Stability of Business Cycles and Trade Links: A Three-country Model with Mixed Type of Fixed and Flexible Exchange Rates

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Abstract

In this study, we focus on the relationship between the stability of business cycle and the degree of trade links, using a three-country model. The Euro area enhance the strength of trade links through customs union and single market, whereas the US implements protectionist policies recently. It is important to analyze what the change of these trade links influence the business cycles of the Euro area and the US. Generally, a two-country model with fixed exchange rate is used when analyzing the Euro area. However, in fact, there is a third country that is connected through flexible exchange rate system with the Euro area. In order to analyze a real international economy, it is necessary to use a three-country model with fixed and flexible exchange rates. Therefore, we investigate the impact of rising trade links in the Euro area on the business cycles of three countries, using a three-country model.

Keyword: Three-country model, Stability of business cycle, Euro area

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1 Introduction

In the field of international macroeconomics, many analyses using a small country model or a two-country model exist. Several studies using a small country model with parfect capital mobility have proved that a fiscal policy is effective and a monetary policy is ineffective under a fixed exchange rate, whereas a fiscal policy is ineffective and a monetary policy is effective under a flexible exchange rate. Arguments of these studies do not hold for two country model with imperfect capital mobility. Asada (2016) proves that not only a fiscal policy but also monetary policy is effective in two-country flexible exchange rate model with imperfect capital mobility. Then, Nakao (2017) shows that not only a monetary policy but also fiscal policy is effective in two-country monetary union (fixed exchange rate) model with imperfect capital mobility.

However, when analyzing the Euro area, the problem is that a two-country model is not sufficient to analyze a "real" international macroeconomy. In real economy, the Euro area form a monetary union, whereas form a flexible exchange rate regimes to United States, Japan, and so on. Therefore, it is necessary to formulate a model included three countries at least to consider a real international economy, notably the Euro area economy. Although Asada (2018) formulates a three country Mundell-Fleming model with imperfect capital mobility and mixed type of fixed and flexible exchange rates, little study has been done to consider a three-country model in other literatures.

The present study investigates what trade links has kind of influence business cycles of three country with imperfect capital mobility and mixed type of monetary union and flexible exchange rate, using a three-country model. Nakao (2018) proves that an increase in the capital mobility between two countries is a destabilizing factor, while a high degree of openness of the economy and a counter-cyclical fiscal policy is stabilizing factor, using a Kaldorian two-country model with a monetary union and imperfect capital mobility. The United States (US) President Trump implements protectionist policies and the Euro area opposes the trade policy of United States. Therefore, the study puts its focus on the effects of this conflict on the business cycles of the Euro area and the United States, using a three-country model as an expanded version of Nakao (2018).

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2 Formulation of the Three-Country Model

$$\dot{Y}_i = \alpha_i \left[C_i + I_i + G_i + J_i - Y_i\right] \ ; \ \alpha_i > 0, \tag{1}$$

$$C_i = c_i \left(Y_i - T_i\right) + C_{0i} \ ; \ 0 < c_i < 1, C_{0i} \geq 0, \tag{2}$$

$$T_i = \tau_i Y_i - T_{0i} \ ; \ 0 < \tau_i < 1, T_{0i} \geq 0, \tag{3}$$

$$I_i = I_i\left(r_i\right) \ ; \ I^i_{r_i} = \frac{\partial I_i}{\partial r_i} < 0, \tag{4}$$

$$G_i = G_{0i} + \gamma_i \left(\bar{Y}_i - Y_i\right) \ ; \ \gamma_i > 0, \tag{5}$$

$$\frac{M_i}{p_i} = L_i\left(Y_i, r_i\right) \ ; \ \frac{\partial L_i}{\partial Y_i} > 0, L^i_{r_i} = \frac{\partial L_i}{\partial r_i} < 0, \tag{6}$$

$$\begin{aligned} J_1 = \delta_U H^U_1\left(Y_1, Y_2\right) + \delta_f H^f_1\left(Y_1, Y_3, E\right) \ ; \ H^{U1}_{Y_1} = \frac{\partial H^U_1}{\partial Y_1} < 0, H^{U1}_{Y_2} = \frac{\partial H^U_1}{\partial Y_2} > 0, H^{f1}_{Y_1} = \frac{\partial H^f_1}{\partial Y_1} < 0, \\ H^{f1}_{Y_3} = \frac{\partial H^f_1}{\partial Y_3} > 0, H^{f1}_{E} = \frac{\partial H^f_1}{\partial E} > 0, 0 \leq \delta_U \leq 1, 0 \leq \delta_f \leq 1, \end{aligned} \tag{7}$$

$$\begin{aligned} J_2 &= \delta_U H^U_2\left(Y_1, Y_2\right) + \delta_f H^f_2\left(Y_2, Y_3, E\right) \\ &= -\delta_U H^U_1\left(Y_1, Y_2\right) + \delta_f H^f_2\left(Y_2, Y_3, E\right) \ ; \ H^{f2}_{Y_2} = \frac{\partial H^f_1}{\partial Y_1} < 0, H^{f2}_{Y_3} = \frac{\partial H^f_1}{\partial Y_3} > 0, H^{f2}_{E} = \frac{\partial H^f_1}{\partial E} > 0, \end{aligned} \tag{8}$$

$$Q_1 = \beta\left\{r_1 - r_2\right\} + \beta\left\{r_1 - r_3 - \frac{\left(E^e - E\right)}{E}\right\} = \beta\left\{2r_1 - r_2 - r_3 - \frac{E^e}{E} + 1\right\} \ ; \ \beta > 0, \tag{9}$$

$$Q_2 = \beta\left\{r_2 - r_1\right\} + \beta\left\{r_2 - r_3 - \frac{E^e - E)}{E}\right\} = \beta\left\{-r_1 - 2r_2 - r_3 - \frac{E^e}{E} + 1\right\}, \tag{10}$$

$$A_1 = J_1 + Q_1, \tag{11}$$

$$A_2 = J_2 + Q_2, \tag{12}$$

$$p_1 A_1 + p_2 A_2 + E p_3 A_3 = 0, \tag{13}$$

$$p_1 J_1 + p_2 J_2 + E p_3 J_3 = 0, \tag{14}$$

$$p_1 Q_1 + p_2 Q_2 + E p_3 Q_3 = 0, \tag{15}$$

$$\dot{M}_1 = A_1, \tag{16}$$

$$\bar{M}_U = M_1 + M_2, \tag{17}$$

$$M_3 = \bar{M}_3, \tag{18}$$

$$A_3 = 0, \tag{19}$$

$$\dot{E}^e = \sigma\left(E - E^e\right) \ ; \ \sigma > 0. \tag{20}$$

where subscript i ($i = 1, 2, 3$) is the index number of a country, and the definitions of the other symbols are as follows: Y_i is real net national income, C_i is real private consumption expenditure, c_i is the marginal propensity to consume, C_{0i} is the basic consumption, I_i is real net private investment expenditure, G_i is real government expenditure, G_{0i} is the basic government expenditure, r_i is the nominal rate of interest,[1)] $\bar{Y}_i$ is the level of real national income that a government determine the counter-cyclical government expenditure (this is not necessarily natural output), T_i is the real income tax, τ_i is the marginal tax rate, T_{0i} is the negative income tax (or basic income), M_i is the nominal money supply, P_i is the price level, L_i is the amount of money demand, $\bar{M}$ is the nominal money supply of supranational central bank, J_i is the real net exports, H^U_i is the potential real net exports capacity to the

[1)] In this study, for the sake of simplicity, public bonds and stock are treated as perfect substitute goods.

partner of a monetary union, H_i^f is the potential real net exports capacity to the partner in flexible exchange rate system, Q_i is the real capital account balance, A_i is the real total balance of payments, E is the exchange rate of the currency of a monetary union per country 3, E^e is the expected exchange rate. The dots above the symbols represent derivatives with respect to time.

Eq. (1) is the disequilibrium quantity adjustment process in the goods market. Parameter α_i represents the adjustment speed of the goods market. Eq. (2) is the Keynesian consumption function indicating the behavior of the consumer. Eq. (3) is the standard tax function. Eq. (4) is the standard Kaldorian investment function. Eq. (5) is the government expenditure function. Parameter γ_i represents the degree of counter-cyclical fiscal policy. The larger γ_i is, the larger is counter-cyclical government expenditure. Eq. (6) represents the equilibrium condition in the monetary market. Eq. (7) is the real net export function of country 1. Eq. (8) is the real net export function of country 2. Parameter δ_U represents the degree of openness of the economy within a monetary union area organized by country 1 and country 2. Parameter δ_f represents the degree of openness of the economy between a monetary union and country 3. Eq. (9) is the real capital account balance function of country 1 in the model with imperfect capital mobility. Eq. (10) is the real capital account balance function of country 2 in the model with imperfect capital mobility. Parameter β indicates the degree of mobility of international capital flows. The larger β is, the higher is the degree of mobility of international capital flows. The model of perfect capital mobility is a special case in which β is infinite, and the following equation is always established in the case of a fixed exchange rate system: $r_1 = r_2$. Eq. (11) is the definitional equation of the real total balance of payments of country 1. Eq. (12) is the definitional equation of the real total balance of payments of country 2. Eq. (13) implies a sum of the total balance of payments of each countries. The same is true of Eq. (14) and (15). Eq. (16) means that the nominal money supply of country 1 increases (decreases) according to the total balance of payment surplus (deficit) of country 1. Eq. (17) indicates that the total nominal money supply of two countries is fixed by the European Central Bank. Eq. (18) indicates that the nominal money supply of country3 is fixed by the central bank of country 3. Eq. (19) means that the exchange rate E between a monetary union and country3 is determined endogenously so that the total balance of payments is balanced. Eq. (20) implies a mechanism on the formulation of expectation about the exchange rate. Parameter σ represents the adjustment speed of the expectation of exchange rate.

Furthermore, we assume a fixed price economy.

$$p_1 = p_2 = p_3 = 1. \tag{21}$$

To simplify the analysis, we focus on a fixed price economy in the short run. This assumption eliminates price fluctuations. Therefore, we do not deal with the issues of inflation and deflation.

Then, we transform this system into a more compact system. We obtain the following LM equation by solving Eq. (6) with respect to r_i.

$$r_i = r_i\left(Y_i, M_i\right)\ ;\ r_{Y_i}^i = \frac{\partial r_i}{\partial Y_i} = -\frac{L_{Y_i}^i}{L_{r_i}^i} > 0, r_{Mi}^i = \frac{\partial r_i}{\partial M_i} = \frac{1}{L_{r_i}^i} < 0. \tag{22}$$

Furthermore, we obtain the following money supply equation of country 2 from Eq. (17).

$$M_2 = \bar{M} - M_1 = M_2\left(M_1\right), \tag{23}$$

Then, we obtain the following equation ofrom Eq. (19).

$$A_1 + A_2 = \delta_f\left(H_1^f\left(Y_1, Y_3, E\right) + H_2^f\left(Y_2, Y_3, E\right)\right) + \beta\left\{r_1\left(Y_1, M_1\right) + r_2\left(Y_1, M_2\left(M_1\right)\right) - 2\left(r_3\left(Y_3, \bar{M}_3\right) + \frac{E^e}{E} - 1\right)\right\}$$
$$= 0. \tag{24}$$

Solving Eq. (24) with respect to E,

$$E = E\left(Y_1, Y_2, Y_3, E^e, M_1\right); \; E_{Y_1} = \frac{\partial E}{\partial Y_1} = -\frac{J^1_{Y_1} + J^2_{Y_1} + \beta r^1_{Y_1}}{J^1_E + J^2_E + 2\beta(E^e/E^2)},$$

$$E_{Y_2} = \frac{\partial E}{\partial Y_2} = -\frac{J^1_{Y_2} + J^2_{Y_2} + \beta r^2_{Y_2}}{J^1_E + J^2_E + 2\beta(E^e/E^2)}, E_{Y_3} = \frac{\partial E}{\partial Y_3} = \frac{-(J^1_{Y_3} + J^2_{Y_3}) + \beta r^3_{Y_3}}{J^1_E + J^2_E + 2\beta(E^e/E^2)},$$

$$E_{E^e} = \frac{\partial E}{\partial E^e} = \frac{2\beta/E}{J^1_E + J^2_E + 2\beta(E^e/E^2)} > 0, E_{M_1} = \frac{\partial E}{\partial M_1} = \frac{\beta(-r^1_{M_1} + r^2_{M_2})}{J^1_E + J^2_E + 2\beta(E^e/E^2)},$$

$$E_{\bar{M}_U} = \frac{\partial E}{\partial \bar{M}_U} = \frac{-\beta r^2_{M_2}}{J^1_E + J^2_E + 2\beta(E^e/E^2)} > 0, E_{\bar{M}_3} = \frac{\partial E}{\partial \bar{M}_3} = \frac{\beta r^3_{\bar{M}_3}}{J^1_E + J^2_E + 2\beta(E^e/E^2)} < 0. \tag{25}$$

Assumption 1 *Because the parameter β is sufficiently large, the following inequations hold.*

$$E_{Y_1} < 0, E_{Y_2} < 0, E_{Y_3} > 0. \tag{26}$$

Then, substituting Eqs. (7), (8) and (21) into Eq. (14), we obtain the following equation by soloving with respect to J_3.

$$J_3 = -\frac{1}{E}\left(J_1 + J_2\right) = -\delta_f \frac{1}{E}\left(H^f_1\left(Y_1, Y_3, E\right) + H^f_2\left(Y_2, Y_3, E\right)\right). \tag{27}$$

Furthermore, substituting Eqs. (9), (10) and (21) into Eq. (15), we obtain the following equation by soloving with respect to Q_3.

$$Q_3 = -\frac{1}{E\left(Y_1, Y_2, Y_3, E^e, M_1\right)}\left(Q_1 + Q_2\right) = \frac{1}{E}\beta\left\{-r_1 - r_2 + 2\left(r_3 + \frac{E^e}{E\left(Y_1, Y_2, Y_3, E^e, M_1\right)} - 1\right)\right\}. \tag{28}$$

Assumption 2 $H^{f1}_{Y_1} + H^{f2}_{Y_1} < 0$ *and* $H^{f2}_{Y_2} + H^{f1}_{Y_2} < 0$.

Combining the above equations, we obtain the following five-dimensional system of nonlinear differential equations.

$$\begin{aligned}\dot{Y}_1 &= \alpha_1\left\{c_1\left(1-\tau_1\right)Y_1 + C_{01} + c_1 T_{01} + G_{01} + \gamma_1\left(\bar{Y}_1 - Y_1\right) + I_1\left(r_1\left(Y_1, M_1\right)\right)\right. \\ &\quad \left. + \delta_U H^U_1\left(Y_1, Y_2\right) + \delta_f H^f_1\left(Y_1, Y_3, E\left(Y_1, Y_2, Y_3, E^e, M_1; \bar{M}_U, \bar{M}_3\right)\right) - Y_1\right\} = F_1\left(Y_1, Y_2, Y_3, E^e, M_1\right),\end{aligned} \tag{29}$$

$$\begin{aligned}\dot{Y}_2 &= \alpha_2\left\{c_2\left(1-\tau_2\right)Y_2 + C_{02} + c_2 T_{02} + G_{02} + \gamma_2\left(\bar{Y}_2 - Y_2\right) + I_2\left(Y_2, r_2\left(Y_2, M_2\left(M_1\right)\right)\right)\right. \\ &\quad \left. - \delta_U H^U_1\left(Y_1, Y_2\right) + \delta_f H^f_2\left(Y_2, Y_3, E\left(Y_1, Y_2, Y_3, E^e, M_1; \bar{M}_U, \bar{M}_3\right)\right) - Y_2\right\} = F_2\left(Y_1, Y_2, Y_3, E^e, M_1\right),\end{aligned} \tag{30}$$

$$\begin{aligned}\dot{Y}_3 &= \alpha_3\left\{c_3\left(1-\tau_3\right)Y_3 + C_{03} + c_3 T_{03} + G_{03} + \gamma_3\left(\bar{Y}_3 - Y_3\right) + I_3\left(Y_3, r_3\left(Y_3\right)\right)\right. \\ &\quad \left. -\delta_f \frac{1}{E\left(Y_1, Y_2, Y_3, E^e, M_1\right)}\left(H^f_1 + H^f_2\right) - Y_3\right\} = F_3\left(Y_1, Y_2, Y_3, E^e, M_1\right),\end{aligned} \tag{31}$$

$$\dot{E}^e = \sigma\left\{E\left(Y_1, Y_2, Y_3, E^e, M_1; \bar{M}_U, \bar{M}_3\right) - E^e\right\} = F_4\left(Y_1, Y_2, Y_3, E^e, M_1\right), \tag{32}$$

$$\begin{aligned}\dot{M}_1 &= \delta_U H^U_1\left(Y_1, Y_2\right) + \delta_f H^f_1\left(Y_1, Y_3, E\left(Y_1, Y_2, Y_3, E^e, M_1\right)\right) \\ &\quad + \beta\left\{2r_1\left(Y_1, M_1\right) - r_2\left(Y_2, M_2\left(M_1\right)\right) - r_3\left(Y_3, \bar{M}_3\right) - \frac{E^e}{E\left(Y_1, Y_2, Y_3, E^e, M_1\right)} + 1\right\} = F_5\left(Y_1, Y_2, Y_3, E^e, M_1\right).\end{aligned} \tag{33}$$

3 Local Stability Analysis

In this section, we assume that a unique equilibrium solution $(Y^*_1, Y^*_2, Y^*_3, E^{e*}, M^*_1) > (0, 0, 0, 0, 0)$ exists, and we analyze the local stability of this equilibrium solution. We can write the Jacobian matrix of the system of

Eqs. (29)–(33) that are evaluated at the equilibrium point.

$$J = \begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} & F_{15} \\ F_{21} & F_{22} & F_{23} & F_{24} & F_{25} \\ F_{31} & F_{32} & F_{33} & F_{34} & F_{35} \\ F_{41} & F_{42} & F_{43} & F_{44} & F_{45} \\ F_{51} & F_{52} & F_{53} & F_{54} & F_{55} \end{bmatrix} = \begin{bmatrix} \alpha_1\Phi_{11} & \alpha_1\Phi_{12} & \alpha_1\Phi_{13} & \alpha_1\Phi_{14} & \alpha_1\Phi_{15} \\ \alpha_2\Phi_{21} & \alpha_2\Phi_{22} & \alpha_2\Phi_{23} & \alpha_2\Phi_{24} & \alpha_2\Phi_{25} \\ \alpha_3\Phi_{31} & \alpha_3\Phi_{32} & \alpha_3\Phi_{33} & \alpha_3\Phi_{34} & \alpha_3\Phi_{35} \\ \sigma\Phi_{41} & \sigma\Phi_{42} & \sigma\Phi_{43} & \sigma\Phi_{44} & \sigma\Phi_{45} \\ F_{51} & F_{52} & F_{53} & F_{54} & F_{55} \end{bmatrix}; \tag{34}$$

$$\Phi_{11} = -\underbrace{\{1-c_1(1-\tau_1)\}}_{(+)} + \underset{(-)}{I^1_{r_1}}\underset{(+)}{r^1_{Y_1}} - \gamma_1 + \delta_U \underset{(-)}{H^{U1}_{Y_1}} + \delta_f \underset{(-)}{H^{f1}_{Y_1}} + \delta_f \underset{(+)}{H^{f1}_E}\underset{(-)}{E_{Y_1}} < 0,\ \Phi_{12} = \delta_U \underset{(+)}{H^{U1}_{Y_2}} + \delta_f \underset{(+)}{H^{f1}_E}\underset{(-)}{E_{Y_2}},$$

$$\Phi_{13} = \delta_f \underset{(+)}{H^{f1}_{Y_3}} + \delta_f \underset{(+)}{H^{f1}_E}\underset{(+)}{E_{Y_3}} > 0,\ \Phi_{14} = \delta_f \underset{(+)}{H^{f1}_E}\underset{(+)}{E_{E^e}} > 0,\ \Phi_{15} = \underset{(-)}{I^1_{r_1}}\underset{(-)}{r^1_{M_1}} + \delta_f \underset{(+)}{H^{f1}_E}\underset{(?)}{E_{M_1}},$$

$$\Phi_{21} = \delta_U \underset{(+)}{H^{U2}_{Y_1}} + \delta_f \underset{(+)}{H^{f2}_E}\underset{(-)}{E_{Y_1}},\ \Phi_{22} = -\underbrace{\{1-c_2(1-\tau_2)\}}_{(+)} + \underset{(-)}{I^2_{r_2}}\underset{(+)}{r^2_{Y_2}} - \gamma_2 + \delta_U \underset{(-)}{H^{U2}_{Y_2}} + \delta_f \underset{(-)}{H^{f2}_{Y_2}} + \delta_f \underset{(+)}{H^{f2}_E}\underset{(-)}{E_{Y_2}} < 0,$$

$$\Phi_{23} = \delta_f \underset{(+)}{H^{f2}_{Y_3}} + \delta_f \underset{(+)}{H^{f2}_E}\underset{(+)}{E_{Y_3}} > 0,\ \Phi_{24} = \delta_f \underset{(+)}{H^{f2}_E}\underset{(+)}{E_{E^e}} > 0,\ \Phi_{25} = -\underset{(-)}{I^2_{r_2}}\underset{(-)}{r^2_{M_2}} + \delta_f \underset{(+)}{H^{f2}_E}\underset{(?)}{E_{M_1}},$$

$$\Phi_{31} = \delta_f \frac{1}{E}\left[-\underbrace{(H^{f1}_{Y_1} + H^{f2}_{Y_1})}_{(-)} + \underset{(-)}{E_{Y_1}}\left\{-(\underset{(+)}{H^{f1}_E} + \underset{(+)}{H^{f2}_E}) + \frac{1}{E}(\underset{(?)}{H^f_1} + \underset{(?)}{H^f_2})\right\}\right],$$

$$\Phi_{32} = \delta_f \frac{1}{E}\left[-\underbrace{(H^{f1}_{Y_2} + H^{f2}_{Y_2})}_{(-)} + \underset{(-)}{E_{Y_2}}\left\{-(\underset{(+)}{H^{f1}_E} + \underset{(+)}{H^{f2}_E}) + \frac{1}{E}(\underset{(?)}{H^f_1} + \underset{(?)}{H^f_2})\right\}\right],$$

$$\Phi_{33} = -\underbrace{\{1-c_3(1-\tau_3)\}}_{(+)} + \underset{(-)}{I^3_{r_3}}\underset{(+)}{r^3_{Y_3}} - \gamma_3 + \delta_f \frac{1}{E}\left[-(\underset{(+)}{H^{f1}_{Y_3}} + \underset{(+)}{H^{f2}_{Y_3}}) + \underset{(+)}{E_{Y_3}}\left\{-(\underset{(+)}{H^{f1}_E} + \underset{(+)}{H^{f2}_E}) + \frac{1}{E}(\underset{(?)}{H^f_1} + \underset{(?)}{H^f_2})\right\}\right],$$

$$\Phi_{34} = \delta_f \underset{(+)}{E_{E^e}}\frac{1}{E}\left\{-(\underset{(+)}{H^{f1}_E} + \underset{(+)}{H^{f2}_E}) + \frac{1}{E}(\underset{(?)}{H^f_1} + \underset{(?)}{H^f_2})\right\},\ \Phi_{35} = -\delta_f \frac{1}{E}(\underset{(+)}{H^{f1}_E} + \underset{(+)}{H^{f2}_E})\underset{(?)}{E_{M_1}},$$

$$\Phi_{41} = \underset{(-)}{E_{Y_1}} < 0,\ \Phi_{42} = \underset{(-)}{E_{Y_2}} < 0,\ \Phi_{43} = \underset{(+)}{E_{Y_3}} > 0,\ \Phi_{44} = \underset{(+)}{E_{E^e}} - 1 = \frac{\beta(2/E)}{\delta_f(\underset{(+)}{H^{f1}_E} + \underset{(+)}{H^{f2}_E}) + \beta(2/E)} - 1 < 0,$$

$$\Phi_{45} = \underset{(?)}{E_{M_1}},\ F_{51} = \delta_f(\underset{(-)}{H^{f1}_{Y_1}} + \underset{(+)}{H^{f1}_E}\underset{(-)}{E_{Y_1}}) + \beta(2\underset{(+)}{r^1_{Y_1}} + \frac{1}{E}\underset{(-)}{E_{Y_1}}),\ F_{52} = \delta_f(\underset{(+)}{H^{f1}_{Y_2}} + \underset{(+)}{H^{f1}_E}\underset{(-)}{E_{Y_2}}) + \beta(-\underset{(+)}{r^2_{Y_2}} + \frac{1}{E}\underset{(-)}{E_{Y_2}}),$$

$$F_{53} = \delta_f(\underset{(+)}{H^{f1}_{Y_3}} + \underset{(+)}{H^{f1}_E}\underset{(+)}{E_{Y_3}}) + \beta(-\underset{(+)}{r^3_{Y_3}} + \frac{1}{E}\underset{(+)}{E_{Y_3}}),\ F_{54} = \delta_f \underset{(+)}{H^{f1}_E}\underset{(+)}{E_{E^e}} + \beta\frac{1}{E}\underbrace{(E_{E^e} - 1)}_{(-)},$$

$$F_{55} = \delta_f \underset{(+)}{H^{f1}_E}\underset{(?)}{E_{M_1}} + \beta(2\underset{(-)}{r^1_{M_1}} + \underset{(-)}{r^2_{M_2}} + \frac{1}{E}\underset{(?)}{E_{M_1}}).$$

Now, we assume as follows.

Assumption 3 *The absolute values of marginal import propensities $H^{Ui}_{Y_i}$ and $H^{fi}_{Y_i}$ are sufficiently large.*

Assumption 4 $\Phi_{12} \simeq 0$, $\Phi_{21} \simeq 0$, $\Phi_{15} > 0$, $\Phi_{25} < 0$, $\Phi_{35} \simeq 0$, $\Phi_{45} \simeq 0$, $F_{55} < 0$, $F_{51} < 0$, $F_{53} > 0$, $F_{52} < 0$, *and* $F_{54} \simeq 0$.

Furthermore, we assume that the following equation hold at the epuilibrium point from Assumptions 4.

Assumption 5

$$\Phi_{12} = \Phi_{21} = \Phi_{35} = \Phi_{45} = \Phi_{54} = 0. \tag{35}$$

Now, we focus on the terms including δ_f in Φ_{31}, Φ_{32}, Φ_{33} and Φ_{34}.

Proposition 1

(i) *Suppose that the parameters α_i, β, γ_i, σ and δ_U are fixed at any level. In addition, suppose that inequalities $\Phi_{31} < 0$, $\Phi_{32} < 0$, $\Phi_{33} > 0$ and $\Phi_{34} > 0$ hold because the potential real net exports capacity of country 3 is negative ($H_1^f + H_2^f > 0$) and the absolute value of the capacity is sufficiently large at the equilibrium point. Then, the equilibrium point of the system (29)–(33) is locally unstable if the parameter δ_f is sufficiently large.*

(ii) *Suppose that the parameters α_i, β, γ_i, σ and δ_U are fixed at any level. In addition, suppose that inequalities $\Phi_{31} > 0$, $\Phi_{32} > 0$, $\Phi_{33} < 0$ and $\Phi_{34} < 0$ hold because the potential real net exports capacity of country 3 is positive ($H_1^f + H_2^f < 0$) and the absolute value of the capacity is sufficiently large at the equilibrium point. Then, the equilibrium point of the system (29)–(33) is locally stable if the parameter δ_f is sufficiently large.*

From Proposition 1, it became clear what protectionist policies influence the business cycles of the Euro area and the US. The US has current account deficit with Euro area, but the US President Trump are about to shift from deficit to surplus, using tariff, quantitative restrictions, tariff quotas, and so on. Propsition 1 (i) indicates that an active trade destabilizes the business cycles of the Euro area and the US, if the current account deficit of US is large. Then, once the US achives current account surplus, it is desireble to increase in trade link. If the US keep current account surplus and protectionist policies, it is not certain whether the business cycles of the Euro area and the US are unstable. However, the power to stabilize the business cycle decrease at least. Therefore, the Euro area need to provide for current account surplus of the US and protectionist policies as follows.

Proposition 2 *Suppose that the parameters α_i, β and σ are fixed at any level and the parameter δ_f is fixed at lower level. In addition, suppose that inequalities $\Phi_{31} > 0$, $\Phi_{32} > 0$, $\Phi_{33} < 0$ and $\Phi_{34} < 0$ hold because the potential real net exports capacity of country 3 is positive ($H_1^f + H_2^f < 0$) and the absolute value of the capacity is sufficiently large at the equilibrium point. Then, the equilibrium point of the system (29)–(33) is locally stable if at least one of the parameters δ_U, γ_1 and γ_2 is sufficiently large.*

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