Subsidy Competition Between Regions:
An Extension to Cross-shareholding and Employment Concerns

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Abstract

This study investigates the relationship between firms' regional location choice and subsidy policy of regional governments in an imperfectly competitive third-market model. We extend the model to the following two notable ways and examine whether the result that no subsidy is given to firms continues to hold. First, we incorporate firms' shareholding across regions into the model and examine whether the difference in the shareholding ratio between shareholders in both regions affects the zero-subsidy result. We demonstrate that even if firms' shareholders exist across the region, the result that regional governments give no subsidy to firms remains unchanged. Second, we consider the situation in which regional governments have concerns about regional employments brought about by a firm located in each region, and examine whether the consideration of employment by governments changes the existing results. When regional governments have concerns about regional employments, there is no equilibrium of subsidy competition.

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1   Introduction

This study investigates how the regional location choice by firms affects the subsidy policy by regional governments in an imperfectly competitive third-market model. In the context of the theory of international trade, Janeba (1998) for the first time showed that when each firm initially located in either home or foreign country can choose its location to produce an export good before competing in exporting to an imperfectly competitive third market, both home and foreign governments impose no export tariff or subsidy on export goods. Its result can be immediately applicable for firm's location choice between regions in the context of regional economies. In other words, when each firm supplying a good in a third market chooses the location to produce after observing subsidy levels by both regional governments, each regional government gives no subsidy to firms.

In this seminal article, Janeba (1998) only explored the model in which regional welfare consists only of the profit of the firm located in the region and did not consider the existence of other stakeholders such as foreign shareholders or employees. However, actually, even if a firm is located in a region, the firm's shareholders might exist across the region and its existence might affect firms' location choice and/or regional governments' subsidy policy. Moreover, aside from shareholders, the existence of employees employed in the firm might also affect their decisions. Generally speaking, social welfare in a region depends not only on the profit of the firm located in a region but also on the welfare of cross-shareholders and/or employees.

Therefore, in this study, we extend the model to the following two notable ways which the existing literature does not explore, and examine whether the result that regional governments give no subsidy to firms continues to hold. First, we incorporate firms' cross-shareholding across regions into the model and examine whether the difference in the shareholding ratio between shareholders in both regions affects the above zero-subsidy result. Second, we consider the situation in which there are employees employed by a firm in each region, and examine whether the existence of employees changes the existing results.

2 The Model

There exist two exporting regions, indexed by region $i \in \{1,2\}$, and an importing region that has a good market. Both exporting regions have no good market and the importing region with a market has no firm to produce the goods. Initially, only one firm is established in region i, which we call firm i, following the indexed region, respectively. Both firm 1 and firm 2 export a homogenous good in the third market and engage in a Cournot duopoly competition. The governments of exporting regions can subsidize the located firms to promote firms' exporting. We henceforth call the government of region i regional government i. The government of the importing region has no active role. q_i denotes firm i's quantity, $Q \equiv q_1 + q_2$ is total output, and p is the price. $p = p(Q)$ is the inverse demand function, assuming $p' < 0$ and $p' + p'' q_i \leq 0$. $C_i(q_i)$ denotes the firm i's cost function and we assume $C_i' > 0$ and $C_i'' \geq 0$. The location choice by firms 1 and 2 is denoted by k and l, $k, l \in \{1,2\}$, respectively. s_k, $k \in \{1,2\}$ denotes regional government k's specific subsidy.

Firms aim to maximize its profit plus export subsidy. Governments aim to maximize its national social welfare. Firm i's profit before subsidizing is $\widehat{\pi}^i \equiv p(Q)q_i - C_i(q_i)$ and firm i's total profit after subsidizing is $\pi^i \equiv \pi^i(q_i, q_j; s_k) = (p(Q) + s_k)q_i - C_i(q_i)$, $j = \{1,2\}$, $j \neq i$, where s_k is the specific export subsidy to firm i by regional government k when firm i locates in region k. Regional governments do not discriminate located firms. Consumer surplus of the third-market region is $CS \equiv \int_0^Q p(x)dx - p(Q)Q$.

We denote the share ratio of firm i's profit for region j's shareholders as α_{ij}, where $\alpha_{ii} \in (0,1]$, $\alpha_{ij} \in [0,1)$, and $\sum_j \alpha_{ij} = 1$. Without loss of generality, it is assumed that $\alpha_{ii} > \frac{1}{2}(> \alpha_{ij})$. Firm i's producer surplus consists of the share of firm i's profit and that of firm j's profit by regional i's residents, that is, $PS_i \equiv \alpha_{ii}\pi^i + \alpha_{ji}\pi^j$. Region i's social welfare is $W_i \equiv PS_i - S_k$. The regional government aims to maximize W_i.

We make several additional assumptions on cost and subsidy. First, the switching cost to change the location is assumed to be zero. Second, we assume that each regional government subsidizes only profits generated in its own region. Third, each firm's cost function is independent of its own and its rival's location choices. Finally, we assume the tie-breaking rule that if there are no subsidy differentials between regions, a firm chooses to locate in the initially established region.

The timing of this three-stage game is described as follows. In the first stage, both regional governments simultaneously and non-cooperatively set the optimal specific subsidy, (s_1, s_2), to maximize the regional social welfare. In the second stage, after correctly observing the optimal subsidy levels, (s_1, s_2), both firms choose the location to produce the export good, (k, l), simultaneously and non-cooperatively. In the third stage, after correctly observing the outcome determined in the first stage and the second stage, both firms engage in a Cournot quantity competition and determine the output, (q_1, q_2), simultaneously and non-cooperatively. Regional governments can commit to their subsidy policy, irrespective of firms' location choice in the second stage. The solution concept is the subgame perfect Nash equilibrium (SPNE). We solve the SPNE by backward induction.

3   When firms' shares hold across regions

3.1   The third stage

Firms 1 and 2 maximize their profit after subsidizing, $\pi^1 = (p(Q) + s_k)q_1 - C_1(q_1)$ and $\pi^2 = (p(Q) + s_l)q_2 - C_2(q_2)$, with respect to q_1 and q_2, respectively. The first-order conditions for profit maximization are as follows:

$$\pi^1_1 = p + p'q_1 - C'_1 + s_k = 0. \tag{3.1}$$

$$\pi^2_2 = p + p'q_2 - C'_2 + s_l = 0. \tag{3.2}$$

We denote the output levels in the equilibrium by $(q_1(s_k, s_l), q_2(s_k, s_l))$, $k, l \in \{1, 2\}$, where k and l denote the regions where firm 1 and firm 2 choose to locate, respectively. The following derivatives are obtained.

$$\frac{\partial q_1}{\partial s_k} = -\frac{\pi^2_{22}}{\Pi} > 0, \quad \frac{\partial q_1}{\partial s_l} = \frac{\pi^1_{12}}{\Pi} \leq 0, \tag{3.3}$$

$$\frac{\partial q_2}{\partial s_l} = -\frac{\pi^1_{11}}{\Pi} > 0, \quad \frac{\partial q_2}{\partial s_k} = \frac{\pi^2_{21}}{\Pi} \leq 0, \tag{3.4}$$

where $\Pi \equiv \pi^1_{11}\pi^2_{22} - \pi^1_{12}\pi^2_{21} > 0$. We obtain the sign of the change in profits as follows:

$$\frac{\partial \pi^1}{\partial s_k} = (1 + \frac{p'\pi^2_{21}}{\Pi})q_1 > 0, \quad \frac{\partial \pi^1}{\partial s_l} = -\frac{p'\pi^1_{11}}{\Pi}q_1 < 0, \tag{3.5}$$

$$\frac{\partial \pi^2}{\partial s_l} = (1 + \frac{p'\pi^1_{12}}{\Pi})q_2 > 0, \quad \frac{\partial \pi^2}{\partial s_k} = -\frac{p'\pi^2_{22}}{\Pi}q_2 < 0. \tag{3.6}$$

3.2   The second stage

In the second stage, firms choose where to locate between regions 1 and 2. Firms prefer higher subsidies and switch their location in a highly subsidized region. Firms choose to shift production completely to the region with higher subsidy. The following lemma is obtained.

Lemma 1. *Each firm chooses to locate in the high-subsidized region.*

Lemma 1 implies that firms always produce in the high-subsidy region when anticipating the result of the Cournot equilibrium in the upcoming third stage.

3.3   The first stage

In the first stage, each regional government set the optimal subsidy rate to maximize its regional welfare, $W_i = \alpha_{ii}\pi^i + \alpha_{ji}\pi^j - S_k$. When firms locate in the same region, the comparative statics of the equilibrium outputs are obtained as follows:

$$q'_1(s) = -\frac{\pi^2_{22} - \pi^1_{12}}{\Pi}, \tag{3.7}$$

$$q'_2(s) = -\frac{\pi^1_{11} - \pi^2_{21}}{\Pi}. \tag{3.8}$$

Throughout this study, we assume the following assumption on what can be called 'limited cost divergence' to exclude any irregular cases.

Assumption 1. Firm's cost function are so similar that $\pi_{ii}^i < \pi_{ji}^j < 0$ holds.

In this assumption, $q_1'(s) > 0$ and $q_2'(s) > 0$. Totally differentiating the profits, we obtain the derivative of profits as follows:

$$\frac{d\pi^1}{ds} = \left(1 + p'q_2'(s)\right)q_1 = \frac{(2p' - C_1'')\pi_{21}^2 - \pi_{11}^1 C_2''}{\Pi}q_1 > 0, \tag{3.9}$$

$$\frac{d\pi^2}{ds} = \left(1 + p'q_1'(s)\right)q_2 = \frac{(2p' - C_2'')\pi_{12}^1 - \pi_{22}^2 C_1''}{\Pi}q_2 > 0. \tag{3.10}$$

Denoting region i's social welfare when firms i and j choose region k and l, respectively, by $W_i^{(k,l)}$, we write down the social welfare of regions 1 and 2 as follows:

$$
\begin{aligned}
W_1^{(1,1)} &= \alpha_{11}\pi^1 + \alpha_{21}\pi^2 - s_1(q_1 + q_2) && \text{if } s_1 > s_2, \\
W_1^{(1,2)} &= \alpha_{11}\pi^1 + \alpha_{21}\pi^2 - s_1 q_1 && \text{if } s_1 = s_2, \\
W_1^{(2,2)} &= \alpha_{11}\pi^1 + \alpha_{21}\pi^2 + s_2 q_1 && \text{if } s_1 < s_2,
\end{aligned}
\tag{3.11}
$$

and

$$
\begin{aligned}
W_2^{(1,1)} &= \alpha_{22}\pi^2 + \alpha_{12}\pi^1 + s_1 q_2 && \text{if } s_1 > s_2, \\
W_2^{(1,2)} &= \alpha_{22}\pi^2 + \alpha_{12}\pi^1 - s_2 q_2 && \text{if } s_1 = s_2, \\
W_2^{(2,2)} &= \alpha_{22}\pi^2 + \alpha_{12}\pi^1 - s_2(q_1 + q_2) && \text{if } s_1 < s_2.
\end{aligned}
\tag{3.12}
$$

Under Assumption 1, we immediately obtain the following lemma.

Lemma 2. *Suppose that Assumption 1 holds and there exists no subsidy in region 2, that is, $s_2 = 0$. Then, the best response for regional government 1 is any nonpositive subsidy rates, irrespective of firms' shareholding ratio across regions. Any subsidy rates from $s_1 \leq 0$ are indifferent for the government.*

Although Lemma 2 is essentially the same as Proposition 3 in Janeba (1998), its claim generally holds even under any firms' cross-sharing ratio between regions. This lemma claims that when there exists no subsidy in both regions initially, neither regional government wishes to attract both firms by subsidization.

Combining Lemmas 1 and 2, we present the main proposition.

Proposition 1. *Suppose that Assumption 1 holds. No subsidy policy for both regional government, $s_1 = s_2 = 0$, is the only SPNE irrespective of the firms' cross-shareholding ratio between regions.*

This proposition claims that even if both firms have the shareholders across regions, the optimal subsidy policy for both regional governments is not to intervene the located firms at all. Stated differently, the firms' cross-shareholding cannot affect the laissez-faire result.

4 When regional governments have concerns about regional employment

We consider the situation in which regional governments have concerns about regional employment and investigate how the regional governments' concerns about regional employment affect their subsidy choices. We assume that firms can elastically demand labor. In addition, we do not consider the cross-shareholding between regions. Labor is the only input to produce a good. L_i^k denotes the labor input by firm i in region k, and w is wage, which is paid per employee. w is assumed to be constant and exogenously given over regions. q_i^k denotes firm i's output located in region k and the identical production function between firms is denoted by $q_i^k = f(L_i^k)$. The labor input function by firm i in region k is denoted by $L_i^k = L(q_i^k)$. We assume $L' > 0$ and $L'' > 0$. Firm i's profit is given as follows:

$$\pi^i = p(Q)q_i - wL_i^k + s_k q_i. \tag{4.1}$$

The first-order conditions for profit maximization are as follows:

$$\pi_1^1 = p + p'q_1 - wL' + s_k = 0. \tag{4.2}$$

$$\pi_2^2 = p + p'q_2 - wL' + s_l = 0. \tag{4.3}$$

Lemma 1 also holds.

In contrast, unlike the previous section, regional governments care about the employment of regional labors. Thus, region i's social welfare consists not only of firm i's profit and subsidy revenue or payment from/to another region, but also the total wage income of employees in region i. That is, $W_i = \pi^i + wL^i - S_i$, where L^i denotes the total employment in region i. As for L^i, the following equalities hold: $L^i = 0$ when no firm locates in region i; $L^i = L_j^i$ when only firm j locates in region i; and $L^i = L_1^i + L_2^i$ when both firms locate in region i.

Region i's social welfare is expressed as follows:

$$
\begin{aligned}
W_1^{(1,1)} &= \pi^1 + w(L_1^1 + L_2^1) - s_1 Q = pq_1 + wL(q_2) - s_1 q_2 & \text{if } s_1 > s_2, \\
W_1^{(1,2)} &= \pi^1 + wL_1^1 - s_1 q_1 = pq_1 & \text{if } s_1 = s_2, \\
W_1^{(2,2)} &= \pi^1 = pq_1 - wL(q_1) + s_2 q_1 & \text{if } s_1 < s_2,
\end{aligned}
\tag{4.4}
$$

and

$$
\begin{aligned}
W_2^{(1,1)} &= \pi^2 = pq_2 - wL(q_2) + s_1 q_2 & \text{if } s_1 > s_2, \\
W_2^{(1,2)} &= \pi^2 + wL_2^2 - s_2 q_2 = pq_2 & \text{if } s_1 = s_2, \\
W_2^{(2,2)} &= \pi^2 + w(L_1^2 + L_2^2) - s_2 Q = pq_2 + wL(q_1) - s_2 q_1 & \text{if } s_1 < s_2.
\end{aligned}
\tag{4.5}
$$

We present the main result on the SPNE.

Proposition 2. *Suppose that Assumption 1 holds and wage is sufficiently large. When both regional governments care about the regional employment, there is no SPNE of subsidy competition.*

The result of Proposition 2 drastically differs from that of Proposition 1. When regional governments do not care about the employment of the regional employees, the laissez-faire result is the unique SPNE. In contrast, when governments have concerns about the regional

employment, there is no SPNE anymore. Proposition 2 suggests that the difference in policy objectives of regional governments causes the different result of subsidy competition between governments. In addition, the nonexistence of the location equilibrium implies that subsidy competition between regional governments is usually instable. Proposition 2 can be interpreted as that both governments fall into the endless subsidy competition to raise subsidy levels.

5 Concluding Remarks

In this study, we reconsidered subsidy competition between regional governments by incorporating other stakeholders of firms into the model. We presented the following results. First, we demonstrated that even if firms' shareholders exist beyond the region, the result that regional governments give no subsidy to firms remains unchanged. This result suggests that any effects of location subsidy by regional governments are invalidated by the ex post firms' location choice, irrespectively of firm's shareholding ratio between regions. Second, we showed that when we extend the basic model to deal with the situation in which regional governments have concerns about regional employment, there is no SPNE of subsidy competition depending on the circumstances. The second result suggests that if regional governments care about regional employees, subsidy competition will be triggered. This result implies that both governments fall into the endless subsidy competition to raise subsidy levels.

References

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