Coordinated state capital tax reform in an overlapping generations model

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Abstract

This paper examines whether a coordinated state capital tax reform improves social welfare under a steady state in an overlapping generations (OLG) model with vertical and horizontal tax externalities. We show that an OLG model adds dynamic effects which do not occur in static models: dynamic efficiency effect and dynamic vertical externality effects. In particular, we can show the sign of dynamic vertical tax externality effect depends on whether each state government ignores the effect of its own tax rate on the federal tax revenue allocate to its own state or not.

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1. Introduction

This paper shows the welfare effects of the coordinated state capital tax reform under vertical and horizontal tax externalities in an overlapping generations model. In particular, we focus on the two cases in which the state governments can consider the effect of its own state tax rate on the federal tax revenue allocated to in its own state and ignore it.

We show that an OLG model adds dynamic effects which do not occur in static models: dynamic efficiency effect and dynamic vertical externality effects. In particular, we can show the sign of dynamic vertical tax externality effect depends on whether each state government ignores the effect on the federal tax revenue allocated to its own state or not. That is, when each state government can recognize this effect, the effect of dynamic vertical externality decreases the state tax rate chosen by the state government. However, when each state government cannot, this dynamic effect increases optimal state capital tax rate.

2. The model

We consider a perfectly competitive economy. The economic activities are carried out in discrete time and last forever. The nation consists of N identical states (indexed by $i = 1, \ldots ., N$). Capital is perfectly mobile and labor is immobile across states. In each state, L_t identical individuals are born in period t and the population is assumed to grow at a rate of n.

There is a single private good produced by using constant returns to scale production technology, $Y_{i,t} = F(K_{i,t}, L_t)$, where $Y_{i,t}$, $K_{i,t}$ and $L_{i,t}$ denote aggregate output, capital input and labor input at states i at period t, respectively. In what follows, we omit subscript of state i except when absolutely necessary. Output per capita can be expressed as $y_t = f(k_t)$, where $y_t \equiv Y_t/L_t$ and $k_t \equiv K_t/L_t$ denote output-labor ratio and capital-labor ratio, respectively.

The profit per capita of firm is given by $f(k_t) - (r_t + \delta + \tau_t)k_t - w_t$, where r_t, δ, w_t and $\tau_t \equiv \tau_t^S + \tau_t^F$ are net interest rate, capital depreciation rate, gross wage rate and a consolidated tax rate, respectively. τ_t^S denotes a state government capital tax rate and τ_t^F does a federal government capital tax rate. The profit maximizing conditions of the firm in perfectly competitive markets are given as:

$$f_k(k_t) = \frac{df(k_t)}{dk_t} = r_t + \delta + \tau_t \equiv R_t, \qquad \qquad (2.1)$$
$$f\big(k_t(R_t)\big) - R_t k_t(R_t) = w_t.$$

From (2.1), we obtain $k_{tR} \equiv dk_t/dR_t = 1/f_{kk} < 0$ and $w_{tR} \equiv dw_t/dR_t = -k_t < 0.$

Individuals live for two periods, the young and the old periods, and both the young and old generations are alive in every period. Those individuals are assumed to be identical both within the same generation and across the different generations. Individuals who are young in period t supply one unit of labor inelastically in exchange for wage income w_{it} and allocate the labor income between consumption in current period c_t and savings s_t. The budget constraint of period t is $w_t = c_t + s_t$. Savings bear the gross rate of return in the next period and enable individuals to consume in the old period. Individuals' consumption in the old period can be represented as $c_{t+1} = (1 + r_{t+1})s_t$. Therefore, the lifetime budget constraint of individuals is given by $w_t = c_t +$

$c_{t+1}/(1+r_{t+1})$.

The utility function for individuals born in period t is given by $u_t(c_t, c_{t+1}) + b(g_{t+1}) + B(G_{t+1})$, where g_t and G_t are the state public goods and the federal public goods available at period t, respectively. $u_t(c_t, c_{t+1})$ is assumed to be additive-separable. The federal public goods provide benefits for all individuals, whereas the state public goods can only benefit residences of the state.

Individuals choose consumptions in both periods to maximize the utility subject to the lifetime budget constraint. From the first-order condition, we obtain the following condition:

$$\frac{\partial u_t/\partial c_t}{\partial u_t/\partial c_{t+1}} = 1 + r_{t+1}, \tag{2.2}$$

From (2.2), savings function is given by $s_t(w_t, r_{t+1})$.We assume that savings function has a following properties, $s_{tw} \equiv \partial s_t/\partial w_t > 0$ and $s_{tr} \equiv \partial s_t/\partial r_{t+1}$. This saving function is assumed to be not decreasing in interest rate: $s_{tr} \geq 0$. Thus the indirect utility function is obtained by

$$\begin{aligned} v_t(w_t, r_{t+1}, g_{t+1}, G_{t+1}) \\ &\equiv u_t\big(w_t - s_t(w_t, r_{t+1}), (1+r_{t+1})s_t(w_t, r_{t+1})\big) + b(g_{t+1}) \\ &\quad + B(G_{t+1}). \end{aligned} \tag{2.3}$$

This indirect utility function has standard properties; $v_{tw} \equiv \partial v_t/\partial w_t > 0$, and $v_{tr} \equiv \partial v_t/\partial r_{t+1} = v_{tw} s_t/(1+r_{t+1})$.

The capital market equilibrium condition at period $t+1$ is given by

$$\sum_{i=1}^{N} s_{i,t}\big(w_{i,t}(R_{i,t}), r_{t+1}\big) = (1+n)\sum_{i=1}^{N} k_{i,t+1}\left(R_{i,t+1}\right). \tag{2.4}$$

This capital market has to satisfy following stability condition:

$$\frac{dr_{t+1}}{dr_t} = \frac{-\sum_{i=1}^{N} s_{iw} w_R}{\sum_{i=1}^{N} s_{ir} - (1+n)\sum_{i=1}^{N} k_{iR}} \in (0,1).$$

We can rewrite stability condition as:

$$\sum_{i=1}^{N}(s_{iw} w_R + s_{ir}) - (1+n)\sum_{i=1}^{N} k_{iR} > 0. \tag{2.5}$$

To prepare for an analysis of the welfare effect by the coordinated state tax reform, we show that comparative statistics about the effect of a changing state capital tax rate on interest rate. As in Keen and Kotsogiannis (2002), the effect of a state capital tax rate is obtained by:

$$\frac{dr}{d\tau_i^S} = -\frac{s_{iw} w_R - (1+n)k_{iR}}{\sum_{i=1}^{N}(s_{iw} w_R + s_{ir} - (1+n)\sum_{i=1}^{N} k_{iR})} \in \left[-\frac{1}{N}, 0\right). \tag{2.6}$$

We assume symmetric equilibrium in which all state governments set the same tax rate. When all state governments coordinately increase tax rates under the symmetric equilibrium, the effect of the coordinated a state capital tax rate is obtained by:

$$\frac{dr}{d\tau^S} = -\frac{s_w w_R - (1+n)k_R}{s_w w_R + s_r - (1+n)k_R} \in [-1, 0). \tag{2.7}$$

Therefore, using (2.6) under symmetric equilibrium, we obtain the following:

$$\frac{dr}{d\tau^S} = N\frac{dr}{d\tau_i^S}. \tag{2.8}$$

The federal government and the state governments supply the federal public goods and the state public goods by spending capital tax revenue, respectively. Thus, the federal government and the state governments face the following budget constraints, respectively:

$$G_t = \frac{1}{N}\tau_t^F\sum_{i=1}^{N} k_{i,t} = \frac{\tau_t^F}{(1+n)N}\sum_{i=1}^{N} s_{i,t-1}(w_{i,t-1}, r_t), \tag{2.9}$$

$$g_{i,t} = \tau_{i,t}^S k_{i,t}. \tag{2.10}$$

We assume that there are no intergovernmental transfers, and the state governments behave as Nash competitors with respect to the federal government and other state governments.

3. State optimal policy rule

In this section, we analyze the state government behavior: the optimal conditions for the state government. Following Batina (2009), the social welfare function used by state government in state i at period t is $W_{i,t} = v_{i,t-1} + v_{i,t}$. In a steady state, this social welfare function can be represented by $W_i = v_i$.[1] In this paper, we focus on only the steady state to show long-run effects. We can consider the following two cases. The first case is that the state governments only consider the effect of its own policy on the federal government revenue allocated to its own state; the second is that the state governments ignore the effect.

3.1 The case where the state governments consider the effect on the federal revenue

First, we consider the case where the state governments take care of the effect of its own policy on the federal government revenue allocated to its own state. In this case, the state governments' problem is formulated by:

$$\max_{\tau_i^S} W_i = v_i(w_i(r+\tau_i), r) + b(g_i) + B(G_i), \quad s.t.\ (2.6), (2.9) \text{ and } (2.10). \tag{2.11}$$

Solving this problem, we obtain the following rule evaluated in symmetric equilibrium:

$$\frac{dW_i}{d\tau_i^S} = -v_w k + \frac{v_w}{1+r}k(n-r)\frac{1}{N}\frac{dr}{d\tau^S} + b_g\left\{k + \tau^S k_R\left(\frac{1}{N}\frac{dr}{d\tau^S}+1\right)\right\} + B_G\tau^F\left\{s_r\frac{1}{N}\frac{dr}{d\tau^S} - s_w k\left(\frac{1}{N}\frac{dr}{d\tau^S}+1\right)\right\} = 0. \tag{2.12}$$

We can see that each state government determines its own state capital tax rate in consideration of the following effects. The first term, $-v_w k$, on the right-hand side in (2.12) represents the effect of the state government's own capital tax rate on wage income through capital accumulation. The second term is dynamic efficiency effect. The third and fourth terms and indicates the effect of the state tax rate

[1] In the steady state, we omit the subscript t.

on the state government's own tax revenue and the federal tax revenue allocated to its own state, respectively.

Dynamic effects are shown in the second term and a part of the fourth term, $-B_G\tau^F s_w k\left(\frac{1}{N}\frac{dr}{d\tau^S}+1\right)$. The other terms except for these terms are static effects. Third term represents static horizontal tax competition effect, and a part of fourth term, $B_{i,G}Ts_r(dr_t/d\tau^{s,i})$, represents static vertical tax competition effect.

The dynamic efficiency effect depends on the dynamic efficiency or inefficiency. If the economy is dynamic efficient, $r > n$, this effect is positive. The other dynamic effect (hereafter, dynamic vertical tax externality effect) means a reduction of federal government's tax base through reduction of wage income due to an increase capital tax rate: the effect on federal governments' tax base through capital accumulation. As we mentioned above, this effect does not appear in static model because saving function depends only on interest rate. This dynamic vertical tax externality effect has welfare effect opposite to the static vertical tax externality effect.

If N is large ($N \to \infty$), we consider the small states as in Batina (2009). In this case, each state government sets an optimal state capital tax rate, ignoring the effect on interest rates: $dr/d\tau^S$ =0. Therefore, (2.12) can be rewritten as $dW_i/d\tau_i^S = -v_w k + b_g\{k+\tau^S k_R\} - B_G\tau^F s_w k = 0$.

3.2 The case where the state governments ignore the effect on the federal revenue

Next, suppose that each state government perfectly ignores the effect of its own tax rate on the federal revenue. In this case, maximization problem of each state government is given by:

$$\max_{\tau_i^S, g_i} W_i = v_i(w_i(r+\tau_i),r) + b(g_i) + B(G_i)\,,\qquad s.t.\ (2.6), (2.10) \text{ and } G_i\ is\ given. \tag{2.13}$$

Solving this problem, we obtain the following condition, evaluated in symmetric equilibrium:

$$\frac{dW_i}{d\tau_i^S} = -v_w k + \frac{v_w}{1+r}k(n-r)\frac{1}{N}\frac{dr}{d\tau^S} + b_g\left\{k+\tau^S k_R\left(\frac{1}{N}\frac{dr}{d\tau^S}+1\right)\right\} = 0. \tag{2.14}$$

Comparing (2.14) with (2.12), the fourth term in (2.12) which implies the effect on the federal revenue does not exist in (2.14). If the sign of the fourth term in (2.12) is positive (negative), the state governments set a lower (higher) state capital tax rate in this section than in section 3.1. Here, we also consider the case where each state is small. In this case, the optimal condition of (2.14) can be written as $dW_i/d\tau_i^S = -v_w k + b_g(k+\tau^S k_R) = 0$.

4. Welfare Effects of Coordinated Capital Tax Reform

In this section, we analyze the effects of a coordinated state capital tax reform on welfare in a steady state. This coordinated tax reform is that all state governments permanently raise their capital tax rate simultaneously, i.e., $d\tau_i^S = d\tau^S > 0$ for all i. The effect of the coordinated tax reform is given by:

$$\frac{dW}{d\tau^S} = -v_w k + \frac{v_w}{1+r} k(n-r)\frac{dr}{d\tau^S} + b_g\left\{k + \tau^S k_R\left(\frac{dr}{d\tau^S}+1\right)\right\}$$
$$+ B_G\tau^F\left\{s_r\frac{dr}{d\tau^S} - s_w k\left(\frac{dr}{d\tau^S}+1\right)\right\}. \tag{2.15}$$

4.1 Coordinated tax reform when the state governments consider the effect on the federal revenue

Firstly, we examine the coordinated tax reform in the situation when state governments consider the effect on the federal revenue. Subtracting (2.12) from (2.15) and using (2.8), we obtain the following result:

$$\frac{dW}{d\tau^S} = \left\{\frac{v_w}{1+r}[k(n-r)] + b_g\tau^S k_R + B_G\tau^F(s_r - s_w k)\right\}\left(1-\frac{1}{N}\right)\frac{dr}{d\tau^S}. \tag{2.16}$$

The coordinated tax reform on welfare can be divided into three effects: the dynamic efficiency effect; the horizontal externality effect; and the vertical externality effect.

Proposition 1.
In overlapping generations model, the effect of the coordinated state capital tax reform on social welfare depends on (1) dynamic efficiency effect, (2) the horizontal externality and (3) the vertical externality effect.

Proposition 2. *Under the situation where capital level satisfies with golden rule,*

(1) if the dynamic vertical externality effect dominates the static vertical externality effect, the coordinated state capital tax reform increases social welfare: the state tax rate is too low relative to the optimal state tax rate.

(2) if the supply of savings is independent of the interest rate, $s_r = 0$, the coordinated state capital tax reform increases social welfare: the state tax rate is too low.

(3) if the demand for capital is independent of the gross interest rate, $k_R = 0$, the welfare effect of coordinated state capital tax reform depends on the dynamic and horizontal vertical externality effects.

4.2 Coordinated tax reform when the state governments ignore the effect on the federal revenue

Next, we consider the case when each state government perfectly ignores the effect on the federal revenue as in Boadway and Keen (1996). Using state government's optimal condition (2.14), we obtain

$$\frac{dW}{d\tau^S} = \left\{\frac{v_w}{1+r}[k(n-r)] + b_g\tau^S k_R\right\}\left(1-\frac{1}{N}\right)\frac{dr}{d\tau^s}$$
$$+ B_G\tau^F\left\{s_r\frac{dr}{d\tau^S} - s_w k\left(\frac{dr}{d\tau^S}+1\right)\right\}. \tag{2.17}$$

In this case, both the static externality effect and dynamic vertical externality effects are definitely negative. This result is inconsistent with that in section 4.1. This is because the each state government ignores the reduction in the federal revenue which is caused by its own tax increase. Thus, the state governments set lower tax rate in this case than in section 4.1.

Proposition 3.
When the each state government perfectly ignores the effect of its own tax rate on the federal revenue, the coordinated tax reform produces the negative vertical externality effect.

Proposition 4. *Under the situation where the each state government perfectly ignores the effect on the federal revenue and capital level satisfies with golden rule,*

(1) the effect of the coordinated state capital tax reform on welfare depends on the horizontal and vertical externality effects.
(2) if the supply of savings is independent of the interest rate, $s_r = 0$, the effect of the coordinated state capital tax reform on welfare depends on the horizontal and vertical externality effects.
(3) if the demand for capital is independent of the gross interest rate, $k_R = 0$, the coordinated state capital tax reform decreases welfare: the state tax rate is too high.

5. Conclusion

The purpose of this paper investigated a coordinated state capital tax reform under vertical and horizontal externalities in an OLG model. We showed that an OLG model adds dynamic effects which do not occur in static models: dynamic efficiency effect and dynamic vertical externality effects. In particular, we can show the direction of dynamic vertical tax externality effect depends on whether each state government ignores the effect of its own tax rate on the federal tax revenue allocate to its own state or not. That is, when each state government can recognize this effect, the coordinated tax reform can produce the positive dynamic effect by vertical externality on welfare as the same direction of horizontal externality: the state tax rate tends to be too low by this dynamic effect. However, when each state government cannot, the coordinated tax reform can produce the negative dynamic effect by vertical externality on welfare as the same direction of static vertical externality: the state tax rate tends to be too high.

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