The effect of Ambient Charges in N-firm Cournot Oligopoly

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1 Introduction

This paper examines the effectiveness of ambient charges which are used as a means of abating emissions of Non-point Source Pollution(NSP). NSP causes serious environmental pollution. Segerson [1] points out that we cannot identify with certainly the source of an observed pollutant or a firm's level of abatement from observations of ambient pollution levels, so mechanisms that focus on ambient pollutant levels rather than emissions are needed in order to control enviromental quality efficiently. He infers to a possible incentive scheme to choose optimal abatement for single or multiple suspected polluters.

Ganguli and Raju[2] explains that since NSP originates from several sources,firm specific emissions are virtually impossible to measure.Ambient charges -charges based on the total amount of pollution irrespective of firm specific origins- constitute one possible mechanism of pollution control. Based on Segerson, Raju and Ganguli, we may indicate that we can make use of ambient charges to abate emissions of NSP.

H.Sato[3] examines that the effectiveness of ambient charges to abate NSP with using two-firm Cournot model. He has suggested that ambient charges could decrease NSP. His model is interesting with introducing Cournot oligopoly model into solving environmental pollution. He provides its solution for only two-firm version. Based on the performance of his model, this paper expands his model to an N-firm version. We may indicate a new expanding model to reduce industrial NSP.

2 H.Sato model

We now consider the effectiveness of environmental policies that use ambient charges as a way to reduce pollutant emissions by the Cournot oligopoly model with two-firm model. In H.Sato model, he analyzes that there are two firms in the same industry and both produce a homogeneous product. In order to demonstrate his model, he makes use of the equations with the Cournot oligopoly model. By using this model, he demonstrates the effectiveness of ambient charges considering two-firm, He shows that a government implements ambient charges to reduce industrial emissions.

Let us now consider his model. The production quantity of firm i,i =1,2 is represented as q_i. As to the market demand function for the products made by these firms, we can obtain

$$p = a - b(q_1 + q_2) \tag{1}$$

where p stands for the price with the products. It is assumed that the production technology with two-firm is the same. a stands for the choke-off price. b shows the marginal costs. Both a and b are positive constants. Firm i,i =1,2 emit pollutants e_iq_i in connection with their products. He shows that it is possible for the government to measure industry's total emissions(i.e., e_1q_1+e_2q_2). The environmental standard $\bar{E}$ is provided exogenously. H.Sato indicates that if e_1q_1+$e_2q_2 \geq \bar{E}$, then the government will levy both firms the same penalty, amounting to m times the difference between the total emission quantity and the environmental standard. Both firms are mutually engaged in Cournot competition, so we can express the profit functions for firm i,i =1,2, as follows;

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$$\pi_i = pq_i - cq_i - m(e_1q_1 + e_2q_2 - \bar{E}) \tag{2}$$

We can reconsider them, as,

$$\pi_i = \Big(a - b(q_1 + q_2) - c - me_i\Big)q_i - m(e_jq_j - \bar{E}), i, j = 1, 2, j \neq i. \tag{3}$$

We transform equation (3) into the equations considering each industry's emission quantity. We can obtain the best response functions with this model, as,

$$BR^i(q_j) = \frac{a - bq_j - c - me_i}{2b}, i, j = 1, 2, j \neq i. \tag{4}$$

me_i expresses the amount of each firms' penalty. With respect to equation (4), we can make the simultaneous equations, as,

$$\begin{cases} a - 2bq_1 - bq_2 - c - me_1 = 0 & \text{(5a)} \\ a - bq_1 - 2bq_2 - c - me_2 = 0 & \text{(5b)} \end{cases}$$

By solving them, we can show the quantities in Cournot equilibrium,

$$(q_1^*, q_2^*) = \left(\frac{a - c - m(2e_1 - e_2)}{3b}, \frac{a - c + m(e_1 - 2e_2)}{3b}\right) \tag{6}$$

When e_1 and e_2 are given, we can indicate the industrial emission $E(m)$ in two-firm model. That is,

$$E(m) = e_1q_1^* + e_2q_2^* = \frac{(a - c)(e_1 + e_2) + 2m(e_1e_2 - e_1^2 - e_2^2)}{3b} \tag{7}$$

This is a function of the policy parameter m, so denote $e_1q_1^*$+$e_2q_2^*$ as a function $E(m)$, differentiating this function by m, gives the following:

$$E'(m) = \frac{2(e_1e_2 - e_1^2 - e_2^2)}{3b} \tag{8}$$

With regard to equation (8), the sign of $E'(m)$ corresponds to the sign of the bracketed part of the numerator(i.e.,e_1e_2-(e_1^2+e_2^2)).Given $e_1>0$ and $e_2>0$, the following proposition holds:

Theorem 1. (Proposition 1 of H.Sato(2016))

$$E'(m) < 0 \tag{9}$$

This indicates that if a government imposes ambient charges into both firms, the amount of total emission quantity in two-firm model will abate.

Proof. H.Sato(2016) analyzes as follows. Given e_1 and e_2, the difference between the square of $(e_1+e_2)/2$ and $\sqrt{e_1e_2}$ is also positive. This is because,

$$\left(\frac{e_1+e_2}{2}\right)^2-(\sqrt{e_1e_2})^2$$
$$=\frac{e_1^2+2e_1e_2+e_2^2}{4}-e_1e_2$$
$$=\frac{e_1^2-2e_1e_2+e_2^2}{4}$$
$$=\frac{(e_1-e_2)^2}{4} \tag{10}$$

Thus, given e_1 and e_2, the following inequality holds:

$$\frac{e_1+e_2}{2}>\sqrt{e_1e_2} \tag{11}$$

When equation(11) are squared, that is,

$$e_1^2+e_2^2>2e_1e_2 \tag{12}$$

Since

$$Sgn\left(e_1e_2-(e_1^2+e_2^2)\right)<0 \tag{13}$$

we have the result as desired.

Q.E.D.(H.Sato(2016))

3 Expansion of H.Sato model

In this section we expand H.Sato(2016) by introducing N-firm version. We examine the effectiveness of ambient charges not only two-firm version but also N-firm version.

Let us now investigate N-firm model. First, we consider the market demand function for the product with N-firm model, as,

$$p=a-b(q_1+q_2+\ldots+q_n) \tag{14}$$

Next, we make the profit function. Considering firm i=n, the profit function with N-firm is given by

$$\begin{aligned}
\pi_i &= pq_i-cq_i-m(e_1q_1+e_2q_2+\cdots+e_nq_n-\bar{E})\\
&=\left(a-b(q_1+q_2+\cdots+q_n)-c-me_i\right)q_i\\
&-m\left((e_1q_1+e_2q_2+\cdots+e_{i-1}q_{i-1})+(e_{i+1}q_{i+1}+\cdots+e_nq_n)-\bar{E}\right), i<n, n\geqq 2\\
&=\left(a-b\sum_{i=1}^{n}q_i-c-me_i\right)q_i-m\left(\sum_{x=1}^{i-1}e_xq_x+\sum_{y=i+1}^{n}e_yq_y-\bar{E}\right)\\
&=\left(a-b\sum_{i=1}^{n}q_i-c-me_i\right)q_i-m\left(\sum_{j=1}^{n}e_jq_j-e_iq_i-\bar{E}\right)\\
&=\left(a-b\sum_{i=1}^{n}q_i-c-me_i\right)q_i-m\left(\sum_{j\neq i}^{n}e_jq_j-\bar{E}\right)
\end{aligned} \tag{15}$$

Differentieing equation (15),

$$\frac{\lambda \pi_i}{\lambda q_i} = a - b\sum_{i=1}^{n} q_i - c - me_i (i = 1, 2, \ldots, n) \tag{16}$$

Equation (16) means the best response functions with N-firm version. Transforming them into the simultaneous functions with N-firm version, as,

$$\begin{cases} a - 2bq_1 - bq_2 - bq_3 - \ldots - bq_i - \ldots - bq_n - c - me_1 = 0 & \text{(17a)} \\ a - bq_1 - 2bq_2 - bq_3 - \ldots - bq_i - \ldots - bq_n - c - me_2 = 0 & \text{(17b)} \\ \vdots & \\ a - bq_1 - bq_2 - bq_3 - \ldots - 2bq_i - \ldots - bq_n - c - me_i = 0 & \text{(17c)} \\ \vdots & \\ a - bq_1 - bq_2 - bq_3 - \ldots - bq_i - \ldots - 2bq_n - c - me_n = 0 & \text{(17d)} \end{cases}$$

Here we express $(17a)$ -$(17d)$ as follows:

$$\mathbf{Aq} = \mathbf{C} \tag{18}$$

$$\mathbf{A} = \begin{pmatrix} 2b & b & b & \ldots & \ldots & b \\ b & 2b & b & \ldots & \ldots & b \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & 2b & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b & b & b & \ldots & \ldots & 2b \end{pmatrix} \qquad q = \begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_i \\ \vdots \\ q_n \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} a - c - me_1 \\ a - c - me_2 \\ \vdots \\ a - c - me_i \\ \vdots \\ a - c - me_n \end{pmatrix}$$

Substituting equation $\mathbf{A}$,q and $\mathbf{C}$ into equation (18) and transforming it, we can obtain

$$q = \mathbf{A^{-1}C} \tag{20}$$

Proof. See the Appendix 1.

Therefore, we can write

$$\begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_i \\ \vdots \\ q_n \end{pmatrix} = \frac{1}{(n+1)b} \begin{pmatrix} n & -1 & -1 & \ldots & \ldots & -1 \\ -1 & n & -1 & \ldots & \ldots & -1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & n & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \ldots & \ldots & n \end{pmatrix} \begin{pmatrix} a - c - me_1 \\ a - c - me_2 \\ \vdots \\ a - c - me_i \\ \vdots \\ a - c - me_n \end{pmatrix} \tag{21}$$

We can consider the industrial function $E(m)$ in N-firm model.

$$E(m) = e_1 q_1^* + e_2 q_2^* + \cdots + e_i q_i^* + \cdots + e_n q_n^* \tag{22}$$

Substituting equation (21) into equation (22), we can obtain equation (23), as,

Theorem 2.

$$E(m) = \frac{1}{(n+1)b}\left[(a-c)\sum_{i=1}^{n} e_i - m\left(\sum_{i=1}^{n-1}\sum_{j=2}^{n}(e_i - e_j)^2 + \sum_{i=1}^{n} e_i^2\right)\right] \tag{23}$$

With respect to the calculation, see the Appendix 2.

Differentiating equation (23) with m,

$$E'(m) = -\frac{1}{(n+1)b}\left(\sum_{i=1}^{n-1}\sum_{j=2}^{n}(e_i - e_j)^2 + \sum_{i=1}^{n} e_i^2\right) \tag{24}$$

n is natural number, so it is postive. b is positive constant by assumption same as two-firm model. $\sum$'s coeffecients are positive, so they are postive, therefore,

Proposition.

$$E'(m) < 0 \tag{25}$$

Equation (25) indicates that when we levy the ambient charges with N firms, the total amount of emission will abate, so we suggest that we can use ambient charges to abete pollutant emission of NSP in N-firm model.

4 Conclusion

We has expanded H.Sato model into N-firm version. We suggest that we could use ambient charges as a way to reduce pollutant emissions in N-firm model. This paper is only static model and not included neither dynamic models nor another incomplete competitive model. It is important for us to consider these model cases because we have to check more types of combinations precisely to express the effectiveness of ambient charges, by which we can refer to the possibility of the extention of this paper model. We can express it in next paper.

References

[1] Segerson, K., 1988. Uncertainty and Incentives for Non-Point Pollution Control. *Journal of environmental economics and management*, 15: 87-98.

[2] Ganguli, S. and Raju,S., 2012. Perverse Environmental Effects of Ambient Charges in a Bertrand Duopoly. *Journal of Environmental Economics and Policy*, 1(3): 289-296.

[3] Sato, H., 2017. Pollution from Cournot Duopoly Industry and the Effect of Ambient Charges. *Journal of Environmental Economics and Policy*, 6(3): 305-308.

Appendix

Appendix 1. Aq=**C**

Proof. First, using martix notation, it may be expressed as

$$\mathbf{A}^{-1} = \begin{pmatrix} x_{11} & x_{12} & x_{13} & \cdots & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & x_{ii} & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & x_{n3} & \cdots & \cdots & x_{nn} \end{pmatrix} \tag{26}$$

Then, multiplying $\mathbf{A}$ by $\mathbf{A^{-1}}$,

$$
\mathbf{A}\circ\mathbf{A^{-1}} = \begin{pmatrix} 2b & b & b & \dots & \dots & b \\ b & 2b & b & \dots & \dots & b \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & 2b & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b & b & b & \dots & \dots & 2b \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} & x_{13} & \dots & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & \dots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & x_{jj} & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n1} & x_{n3} & x_{nj} & \dots & x_{nn} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \dots & \dots & 0 \\ 0 & 1 & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & 1 & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \dots & 1 \end{pmatrix} \tag{27}
$$

Considering row n and column j, this equation can be rewritten as follows.

$$
\begin{cases}
2bx_{1j} + bx_{2j} + bx_{3j} + \ldots + bx_{nj} = 0 & \text{(28a)} \\
bx_{1j} + 2bx_{2j} + bx_{3j} + \ldots + bx_{nj} = 0 & \text{(28b)} \\
bx_{1j} + bx_{2j} + 2bx_{3j} + \ldots + bx_{nj} = 0 & \text{(28c)} \\
\vdots & \\
bx_{1j} + bx_{2j} + \ldots + 2bx_{jj} + \ldots + bx_{nj} = 0 & \text{(28d)} \\
\vdots & \\
bx_{1j} + bx_{2j} + bx_{3j} + \ldots + 2bx_{n-1j} + bx_{nj} = 0 & \text{(28e)} \\
bx_{1j} + bx_{2j} + bx_{3j} + \ldots + bx_{n-1j} + 2bx_{nj} = 1 & \text{(28f)}
\end{cases}
$$

Here we subtract $(28a)$ from $(28b)$. It is

$$
bx_{1j} - bx_{2j} = 0 \tag{29}
$$

Calculating it,

$$
x_{1j} = x_{2j} \tag{30}
$$

We subtract $(28b)$ from $(28c)$, That is,

$$
bx_{2j} - bx_{3j} = 0 \tag{31}
$$

Calculating it,

$$
x_{2j} = x_{3j} \tag{32}
$$

Considering these calculations, we can express the simultaneous equations between (n-1) and n.,as,

$$
x_{n-1j} = x_{nj}, (n = 1, \ldots .i) \tag{33}
$$

Equation (30), (32) and (33) are equal, so we hold

$$
x_{1j} = x_{2j} = \ldots = x_{n-1j} = x \tag{34}
$$

Substituting equation (34) into equation (28a)-(28d), we can obtain the simultaneous equations,as,

$$
\begin{cases}
bx(n-2) + 2bx + bx_{jj} = 0 & \text{(35a)} \\
bx(n-1) + 2bx_{jj} = 1 & \text{(35b)}
\end{cases}
$$

transforming these equations, as,

$$
\begin{cases}
bnx + bx_{jj} = 0 & \text{(36a)} \\
b(n-1)x + 2bx_{jj} = 1 & \text{(36b)}
\end{cases}
$$

Now we can obtain,

$$x = x_{nj} = -\frac{1}{(n+1)b} \tag{37}$$

Substituting equation(37) into (36b),

$$x_{jj} = \frac{n}{(n+1)b} \tag{38}$$

Q.E.D.

Appendix 2.

$$\begin{aligned}
E(m) &= \frac{1}{(n+1)b}\Bigg[(a-c)(e_1+e_2+\cdots+e_n) - m\Bigg(e_1(ne_1-e_2-\cdots-e_n)+e_2(e_1-ne_2-\cdots-e_n)\\
&\quad +\cdots+e_n(e_1-e_2-\cdots-ne_n)\Bigg)\Bigg]\\
&= \frac{1}{(n+1)b}\Bigg[(a-c)(e_1+e_2+\cdots+e_n) - m\Bigg(ne_1^2-e_1e_2-e_1e_3-\cdots-e_1e_n\\
&\quad +ne_2^2-e_2e_1-e_2e_3-\cdots-e_2e_n+\cdots+ne_n^2-e_ne_1-e_ne_2-\cdots-e_ne_{n-1}\Bigg)\Bigg]\\
&= \frac{1}{(n+1)b}\Bigg[(a-c)(e_1+e_2+\cdots+e_n) - m\Bigg((e_1^2-2e_1e_2+e_2^2)+(e_1^2-2e_1e_3+e_3^2)+\dots\\
&\quad +\cdots+(e_1^2-2e_1e_n+e_n^2)+(e_2^2-2e_2e_3+e_3^2)+(e_2^2-2e_2e_4+e_4^2)+\cdots+(e_2^2-2e_2e_n+e_n^2)\\
&\quad +\cdots+(e_{n-1}^2-2e_{n-1}e_n+e_n^2)+(e_1^2+e_2^2+\cdots+e_n^2)\Bigg)\Bigg]\\
&= \frac{1}{(n+1)b}\Bigg[(a-c)(e_1+e_2+\cdots+e_n) - m\Bigg((e_1-e_2)^2+(e_1-e_3)^2+\cdots+(e_1-e_n)^2\\
&\quad +(e_2-e_3)^2+(e_2-e_4)^2+\cdots+(e_2-e_n)^2+\cdots+(e_{n-1}-e_n)^2+\sum_{i=1}^{n}e_i^2\Bigg)\Bigg]
\end{aligned} \tag{39}$$

Q.E.D.