Mixed Education System and Inequality

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Abstract

We study a simple growth model in which public education and private education coexist. Public schools are financed through an income tax and provide a uniform level of education for students. Parents can choose a level of private education for their children and pay the cost by themselves. Parents make their children receive either public or private education. Since the level of public education for a student is fixed, the government needs less revenue when the number of students who attend public school declines. This decline lowers the tax rate and is more beneficial for the parents who make their children attend private schools than those who make their children attend public schools. The convers is also true. This mechanism leads to multiple equilibria of educational choices by parents. The simplified setup enables us to analyze dynamics and mobility in educational choices.

1 Introduction

In this paper, we analyze a growth model with the mixed education system and examine its implications for economic development and inequality. There is the vast literature on economic growth and education. Since Lucas (1988), human capital accumulation is viewed as one of main factors which affect economic growth and development. His idea is applied to various analyses. Among others, Glomm and Ravikumar (1992) compare the public and private education systems and examine their effects on economic development and inequality. Under the public education system, the government imposes an income tax and allocates the revenue for education. Under the private education system, parents choose the level of educational expenditure for their children.

In this paper, we analyze the mixed education system, under which parents can choose either public or private education for their children. All parents pay income taxes. When a parent chooses the public education, she does not pay tuition. In contrast, when a parent chooses the private education, she must pay education costs in addition to an income tax. The public education provides students with a fixed and uniform level of education. Parents can choose freely the level of private education. When a parent is sufficiently rich and is not satisfied with the level of public education, she chooses private education. Because the level of per capita public educational expenditure is fixed, the government

needs to collect more revenue and raise the tax rate when the number of students who attend the public school increases. This change in the tax rate may affect the educational behaviors of parents. Specifically, the rise in the tax rate is more harmful to private education than to public education, and thus induces more parents to choose public education. In sum, the increase in the number of parents who choose public education for their children induces parents to choose public education. The converse is also true. As a resulut, multiple equilibria may occur. In the model, there are two types of agents: the high-skilled and the low-skilled. The above mechanism applies to each type of agents, and three types of equilibria can emerge: an equilibrium in which all parents choose public education, an equilibrium in which all parents choose private education, and an equilibrium in which high-skilled parents choose private education and low-skilled parents choose public educaiton.

We also examine dynamic implications of the model. Because the per capita educational cost is constant, if the number of students who attend the public school does not change, economic growth lowers the burden of public education. An increase in aggregate production makes it possible to lower the income tax rate to finance the total cost of public education, which is constant. As noted above, the decline in the income tax rate induces parents to choose private education. That is, in a rich country, parents tend to choose private education. This effect can generate interesting dynamic process. Suppose that an economy is unequal and that high-skilled individuals receive private education and low-skilled individuals receive public education in the early stage of development. As the economy grows, the income tax rate declines. When the economy reaches a level of development, low-skilled individuals may start to choose private education. After that, the economy realizes a high level of development and equality. In another scenario, the economy may be trapped in a low level of development and remain unequal. The simplified setup of the model enables us to analyze the complicated development process.

The assumption that the per capita public educational expenditure is constant is crucial in the model. It reflects following considerations. First, education has its minimum and requisite contents, such as literacy and numeracy. Teaching alphabets from A to S is useless. Second, the government must provide students with minimum knowledge and skills which are necessary for daily life and working in the economy, but is not required to provide additional and higher knowledge and skills. In other words, a range of choices in the level of public education is narrow. The assumption that the per capita public educational expenditure is constant is an extreme way to reflect these considerations. Needless to say, in reality, it is possible that the governments of poor economies can not collect sufficient revenue to provide students with the minimum level of education. Therefore, the analysis of the model applys to economies which are not so poor.

The remainder of the paper is organized as follows. Section 2 describes the basic setup of the economy. Section 3 considers households' decisions. Section 4 illustrates equilibria. Section 5 analyzes dynamics of the economy. Section 6 concludes.

2 Model

Consider an over-lapping generations economy where individuals live for two periods. There are two types of individuals in the economy, high skilled and low skilled, which are distinguished by the level of their human capital. The number of high skilled individuals in period t is H_t and low skilled L_t, where $H_t + L_t = 1$. Each individual has a single parent and a single child, so the population is constant over time. In the first period, childhood, each individual receives education. This educational choice is made by her parent. Young individuals make no economic choice. In the second period, adulthood, each individual gives birth to one child and is endowed with one unit of time, which she allocates between labor and leisure. Her income is allocated to her consumption and may be allocated to expenditure on the education of her child.

There are two different systems of education in the economy. One is a private school where a parent can freely choose the level of education and must pay the corresponding education cost. The other is a public school which provides a uniform level of education to students. The public school is financed through an income tax. So the parent of the child attending the public school do not pay tuition. Parents must choose between the public school and the private one and makes their children attend it.

The utility function of an individual of generation t is given by:

$$U_t = (1 - \beta_1 - \beta_2) \ln (1 - n_{t+1}) + \beta_1 \ln c_{t+1} + \beta_2 \ln e_{k,t+1}, \tag{1}$$

where n_{t+1} is the fraction of time spent working of an individual of generation t in period $t+1$, c_{t+1} is the consumption of an individual of generation t in period $t+1$, $\beta_1 > 0$ is the weight attached to her consumption, and $\beta_2 > 0$ is the relative weight to the level of her child's education. $e_{k,t+1}$ is the level of education that her child receives, where $k = r, u$ represents the type of education, private or public. $\beta_2 \ln e_{k,t+1}$ reflects altruism factor which is referred to as joy of giving. We assume that $\beta_1 + \beta_2$ is less than 1 and that $1 - \beta_1 - \beta_2$ is the weight attached to leisure. The level of her child's education depends on her education choices.

$$e_{k,t+1} = \begin{cases} e_u & \text{public education} \\ e_{r,t+1} & \text{private education} \end{cases},$$

where e_u is the constant level of public education. When a parent decides to make her child receives private education, she chooses the level of education, $e_{r,t+1}$. The budget constraint of an individual of generation t:

$$c_{t+1} + e_{r,t+1} = (1 - \tau_{t+1}) \, y_{t+1}, \tag{2}$$

where y_{t+1} is income of a member of generation t, which depends on the level of education $e_{k,t}$ she received in her youth and the fraction of time spent on working, n_{t+1}. We adopt a specific functional form of y_{t+1} as

$$y_{t+1} = A e_{k,t}^{\alpha_1} n_{t+1}^{\alpha_2} \tag{3}$$

where $A > 0$, $0 < \alpha_1, \alpha_2 < 1$.

Public education is financed by a proportinal tax, τ_{t+1}, on income. The government budget constraint is

$$N_{t+1} e_u = \tau_{t+1} Y_{t+1}, \tag{4}$$

where N_{t+1} is the number of students who attend the public school and Y_{t+1} is aggregate income. Parents choose either public school or private one to make their children enter, so N_{t+1} is a variable. The public school offers a basic level of education which is necessary to live in the society, so e_u is a constant. The government cannot control aggregate income, so the tax rate, τ_{t+1}, is adjusted to equate expenditure and revenue every period.

3 Household Decision

A parent chooses leisure, consumption and a education system for her child. First, we consider the optimization of the parent who chooses the private school.

3.1 When a parent chooses private school

We can rewrite the utility function (1):

$$U_t^r = (1-\beta_1-\beta_2) \ln (1 - n_{t+1}) + \beta_1 \ln \left[(1 - \tau_{t+1}) y_{t+1} - e_{r,t+1} \right] + \beta_2 \ln e_{r,t+1}, \tag{5}$$

where r denotes private school. The first-order condition for a maximum U_t^r with respect to the level of education $e_{r,t+1}$ leads to:

$$e_{r,t+1} = \frac{\beta_1}{\beta_1 + \beta_2} (1 - \tau_{t+1}) y_{t+1}. \tag{6}$$

Education spending by a parent is a constant fraction of her disposable income.

Next, the first-order condition with respect to the fraction of time spent working n_{t+1} leads to:

$$\frac{\frac{\partial y_t}{\partial n_{t+1}}}{y_t} = \frac{1 - \beta_1 - \beta_2}{\beta_1 + \beta_2} \frac{1}{1 - n_{t+1}}. \tag{7}$$

Substituting the production function (3) into (7) leads to the optimal fraction of time spent working:

$$n_r = \frac{\alpha_2}{\left(\frac{1-\beta_1-\beta_2}{\beta_1+\beta_2} + \alpha_2 \right)} \tag{8}$$

We find that when the parents who choose the private school the optimal fraction of time spent working is constant over time.

3.2 When a parent chooses public school

Now, consider the optimization of the parents who choose the public school. The gorvenment provides the common level of education to students. Hence, a parent does not choose an individual education level for her child. We can rewrite the utility function (1):

$$U_t^u = (1-\beta_1-\beta_2)\ln(1-n_{t+1}) + \beta_1 \ln(1-\tau_{t+1})\, y_{t+1} + \beta_2 \ln e_u, \tag{9}$$

where u denotes public school. The first-order condition for a maximum U_t^u with respect to the fraction of time spent working n_{t+1} gives us:

$$n_u = \frac{\alpha_2}{\left(\frac{1-\beta_1-\beta_2}{\beta_1} + \alpha_2\right)}. \tag{10}$$

This is constant and smaller than the fraction of time spent working when a parent chooses a private school, that is,

$$n_u < n_r.$$

The reason for this difference is as follows. Parents choose the time spent working so as to equate marginal benefit of working with marginal cost of working. When the parents choose the public school marginal benefit of working is *only* due to the increase of consumption. On the other hand, when they choose the private school marginal benefit of working is due to the increase of consumption *plus* the increase of education level. Marginal costs of working are the same between the two. Thus, a parent works more when she sends her child to a private school than to a public one.

Given the level of education that they received when young, the incomes of the parents who choose the private school is higher than that of the parents who choose the public school because of the difference of working time. That is,

$$y_{t+1}^u < y_{t+1}^r.$$

3.3 Educational Choice

A parent chooses a private (public) school to make her child attend if her utility from a private school, U_t^r, is greater (smaller) than her utility from a public school, U_t^u. Substituting both (6) and (8) into (5) yields:

$$\begin{aligned} U_{j,t}^r = {} & (1-\beta_1-\beta_2)\ln(1-n_r) \\ & + \ln\left[\left(\frac{\beta_1}{\beta_1+\beta_2}\right)^{\beta_1}\left(\frac{\beta_2}{\beta_1+\beta_2}\right)^{\beta_2}\left[(1-\tau_{t+1})\, y_{j,t+1}^r\right]^{\beta_1+\beta_2}\right], \end{aligned} \tag{11}$$

where $j = r, u$ represents the type of education the parent received when young. Similarly, substituting (10) into (9) yields:

$$U_{j,t}^u = (1-\beta_1-\beta_2)\ln(1-n_u) + \beta_1 \ln\left[(1-\tau_{t+1})\, y_{j,t+1}^u\right] + \beta_2 \ln e_u. \tag{12}$$

We define a function Φ by using equations (11) and (12):

$$\Phi_{j,t} \equiv U^r_{j,t} - U^u_{j,t} \tag{13}$$

$$= (1-\beta_1-\beta_2)\ln\frac{1-n_r}{1-n_u} + (\beta_1+\beta_2)\ln y^r_{j,t+1} + \beta_2\ln(1-\tau_{t+1}) - \beta_1\ln y^u_{j,t+1} - \beta_2\ln e_u + \ln Z, \tag{14}$$

where the constant Z is defined as

$$Z \equiv \left(\frac{\beta_1}{\beta_1+\beta_2}\right)^{\beta_1}\left(\frac{\beta_2}{\beta_1+\beta_2}\right)^{\beta_2}.$$

The value of $\Phi_{j,t}$ represents utility premium of private education. If it is positive for a parent, she chooses to make her child receive private education. We can see that a rise in τ_{t+1} decreases $\Phi_{j,t}$. If a parent makes her child receive private education, she must decrease expenditures on consumption and education in response to a rise in the income tax rate. She sustains great damage. By contrast, a parent who makes her child receive public education must decrease expenditure only on consumption when the tax rate rises. The level of public education does not change. So she sustains a little damage by the rise in the tax rate. As a result, it decreases utility premium of private education. The value of τ_{t+1} depends on the number of students who receive public education. In the next section, we analyze $\Phi_{j,t}$ and find the equilibrium number of students who receive public and private education.

4 Equilibrium

We assume that all rich parents received private education when young and that all poor parents received public education when young in the initial period. In other words, the economy is unequal.

We denote the number of students who receive public education and whose parents received private education by m^d, and the number of students who receive private education and whose parents received public education by m^u, where $0 \le m^d \le H_t$ and $0 \le m^u \le L_t$. That is, m^d and m^u represent the number of children who receive the different type of education from their parents. We can interpret them as intergenerational mobility: m^d as downward mobility and m^u as upward mobility. Then, the number of students who receive public education, N_{t+1}, is written as

$$N_{t+1} = m^d_{t+1} + (L_t - m^u_{t+1}). \tag{15}$$

And aggregate production can be written as

$$Y_{t+1} = (H_t - m^d_{t+1})y^r_{r,t+1} + m^d_{t+1}y^u_{r,t+1} + (L_t - m^u_{t+1})y^u_{u,t+1} + m^u_{t+1}y^r_{u,t+1}. \tag{16}$$

Therefore, the income tax rate is determined as

$$\tau_{t+1} = \frac{(m^d_{t+1} - m^u_{t+1})e_u + L_t e_u}{H_t y^r_{r,t+1} + L_t y^u_{u,t+1} - m^d_{t+1}(y^r_{r,t+1} - y^u_{r,t+1}) + m^u_{t+1}(y^r_{u,t+1} - y^u_{u,t+1})}. \tag{17}$$

$y^k_{j,t+1}$ is production by a parent, the type of education that she received when young is denoted by $j = r, u$. It is clear that $\partial\tau_{t+1}/\partial m^d_{t+1} > 0$ and $\partial\tau_{t+1}/\partial m^u_{t+1} < 0$. In words, a rise in the number of students who receive public education raises the income tax rate.

Now, we can analyze the parental choice between private and public education and derive the equilibrium number of students. In the initial period, there are H_t high-skilled parents, who received private education when young, and m^d_{t+1} of them decide to make her children receive public education. There are L_t low-skilled parents, who received public education when young, and m^u_{t+1} of them decide to make her children receive private education. We need to determine the values of m^d_{t+1} and m^u_{t+1} in equilibrium. If $\Phi_{r,t} > (<)\ 0$ in equilibrium, it is not rational for a high-skilled parent to choose public (private) education for her child, and thus m^d_{t+1} must be equal to zero (H_t). Similar consideration can be applied to the choice by a low-skilled parent.

In order to find candidates of equilibrium, we depict $\Phi_{j,t} = 0, j = r, u$ loci on (m^d_{t+1}, m^u_{t+1}) plane. The functions $\Phi_{r,t} = 0$ and $\Phi_{u,t} = 0$ have the different constants and have the same functional form except the constants. So they do not intersect. They have the derivative,

$$\frac{dm^d_{t+1}}{dm^u_{t+1}} = -\frac{\partial\tau}{\partial m^u_{t+1}} \Big/ \frac{\partial\tau}{\partial m^d_{t+1}} > 0,$$

so they have positive slopes. From (14) and (17), we can get

$$\frac{\partial\Phi_{r,t}}{\partial m^d_{t+1}} < 0 \quad \text{and} \quad \frac{\partial\Phi_{u,t}}{\partial m^u_{t+1}} > 0.$$

Thus, m^d_{t+1} and m^u_{t+1} are adjusted in the directions indicated by the arrows in Figure 1. There are three candidates for equilibria. Point e corresponds to downward-mobility equilibrium. That is, all high-skilled parents, who received private education when young, choose public education for their children and all low-skilled parents, who received public education, choose public education for their children. Point g corresponds to upward-mobility equilibrium. All parents choose private education for their children. Point f corresponds to no-mobility equilibrium. All parents choose the same type of education for their children as they received when young. In equilibrium, parents who belong to the same income group make the same educational choice.

We can see from Figure 1 that the positions of $\Phi_{r,t} = 0$ and $\Phi_{u,t} = 0$ loci change the combination of equilibria. If $\Phi_{r,t} = 0$ locus intersects the vertical axis at $m^d_{t+1} > H_t$, as in Fig 1E, then downward-mobility equilibrium does not exist. If it intersects the horizontal axis at $m^u_{t+1} > 0$, as in Fig 1F, then no-mobility equilibrium does not exist. An increase in y^r_{t+1} shifts $\Phi_{r,t} = 0$ locus upward because $\partial\Phi_{r,t}/\partial y^r_{r,t+1} > 0$ and $\partial\Phi_{r,t}/\partial m^d_{t+1} < 0$. That is, as the incomes of the families whose members receive private education increase, it is less likely that the downward-mobility equilibrium exists.

If $\Phi_{u,t} = 0$ locus intersects the horizontal axis at $m^u_{t+1} > L_t$, as in Fig 1F, then upward-mobility equilibrium does not exist. If it intersects vertical axis

at $m^d_{t+1} > 0$, as in Fig 1D and 1E, no-mobility equilibrium does not exist. An increase in productivity parameter A shifts $\Phi_{u,t} = 0$ (and $\Phi_{r,t} = 0$) loci upward. That is, when the productivity is sufficiently high, upward-mobility equilibrium is likely to emerge, as in Fig 1C, 1D, and 1E.

5 Dynamics

There are three types of dynamic process without mobility. First, all students receive public education. Second, all students receive private education. Third, students from rich households receive private education and students from poor households receive public education.

5.1 Case 1 - all students receive public education

Because both the level of public education, e_u, and the effort level of an agent who choose public education, n_u, are constant, production level,

$$y^u_{u,t+1} = Ae^{\alpha_1}_u n^{\alpha_2}_u, \tag{18}$$

is also constant. So inequality in income vanishes once all students receive public education.

In this case, privately-educated parents will not exist next period, that is $H_{t+1} = 0$, and thus $\Phi_{r,t} = 0$ locus does not matter on (m^d_{t+1}, m^u_{t+1}) plane. Where the $\Phi_{u,t} = 0$ locus intersects with the horizontal axis determines which type of equilibrium exists. As noted above, the $\Phi_{u,t} = 0$ locus shifts as productivity changes. When productivity is low, only no-mobility equilibrium exists. As productivity rises, upward-mobility equilibrium emerges. When productivity rises further, no-mobility equilibrium disappears. Then, all parents choose private education for their children.

Proposition 1 *When all parents choose public education for their children in a period (i.e. the downward-mobility equilibrium materializes), all descendants will have the same level of income and equality of income will be realized from next period onward.*

5.2 Case 2 - all students receive private education

The production level of an individual who received private education and choose private education for her child is given by

$$y^r_{i,t+1} = A\left(\frac{\beta_1}{\beta_1 + \beta_2} y^r_{i,t}\right)^{\alpha_1} n^{\alpha_2}_r, \quad i = h,\ l. \tag{19}$$

No one receives public education, so the income tax rate is zero. We can see from (19) that the production levels of both high-skilled and low-skilled households converge to

$$y^r_{ss} = \left[A\left(\frac{\beta_1}{\beta_1 + \beta_2}\right)^{\alpha_1} n^{\alpha_2}_r\right]^{\frac{1}{1-\alpha_1}} \tag{20}$$

in the steady state. As a result, inequality in income vanishes in the long run.

In this case, there are no households whose child receive public education, $L_t = 0$, and thus $\Phi_{u,t} = 0$ locus does not matter on (m^d_{t+1}, m^u_{t+1}) plane and $\Phi_{r,t} = 0$ locus intersects the vertical axis at $m^d_{t+1} > 0$. Both an increase in $y^r_{i,t+1}$ and a rise in productivity shift $\Phi_{r,t} = 0$ locus upward. Therefore, downward-mobility equilibrium will disappear over time.

Proposition 2 *When all parents choose private education for their children in a period (i.e. the upward-mobility equilibrium materializes), income inequality will diminish gradually from next period on and vanish in the long run.*

5.3 Case 3 - the high-skilled choose private education and the low-skilled choose public education

In no-mobility equilibrium, production level of the households which choose public education is constant, $y^u_{u,t+1} = Ae^{\alpha_1}_u n^{\alpha_2}_u$. And production level of the households which choose private education is given by

$$y^r_{r,t+1} = A\left[\frac{\beta_1}{\beta_1+\beta_2}(1-\tau_t)y^r_{r,t}\right]^{\alpha_1} n^{\alpha_2}_r, \tag{21}$$

where the income tax rate is determined as

$$\tau_t = \frac{L_{t-1}e_u}{H_{t-1}y^r_{r,t} + L_{t-1}y^u_{u,t}}$$

in this case. Thus, equation (21) is an increasing and concave function of $y^r_{r,t}$ and has a steady state $y^r_{r,t+1} = y^r_{r,t} = y^r_{r,ss}$.

From (18), (19) and (21), we can compare the aggregate level of production. In case 1, all individuals spend less money on education and less time on work than in case 2. Some individuals spend more money on education and more time on work than other individuals in case 3. We can conclude:

Proposition 3 *The aggregate level of production is the largest in case 2., and the smallest in case 1.*

5.4 Development Process

In the previous subsection, we assume that the economy remains in no-mobility equilibrium. But this may not occur. As $y^r_{r,t}$ increases, $\Phi_{r,t} = 0$ and $\Phi_{u,t} = 0$ loci move upward. Exogenous improvement in technology A also moves both upward. Suppose that, in a level of development, the economy is in the situation depicted in Fig 1C and no-mobility equilibrium (f) is realized. As the economy develops, $\Phi_{r,t} = 0$ and $\Phi_{u,t} = 0$ loci move upward. As a result, the situation may change into Fig 1D or 1E. Then, no mobility is no longer an equilibrium. Some parents must change their behaviors. The more probable result is that upward mobility occurs. That is, parents who received public education choose private education for their children. After that, the economy is in case 2 and converges to the steady state with the high aggregate level of production.

6 Conclusion

In this paper, implications of the mixed education system are examined. Parents choose either public or private education for their children. The government collects income taxes and provides public education. The income tax rate is adjusted through the government budget constraint, and the adjustment may generate multiple equilibria. In the long run, the economy may be trapped in the unequal and low production steady state. Also it is possible that the economy arrives at the equal and high production steady state.

Reference

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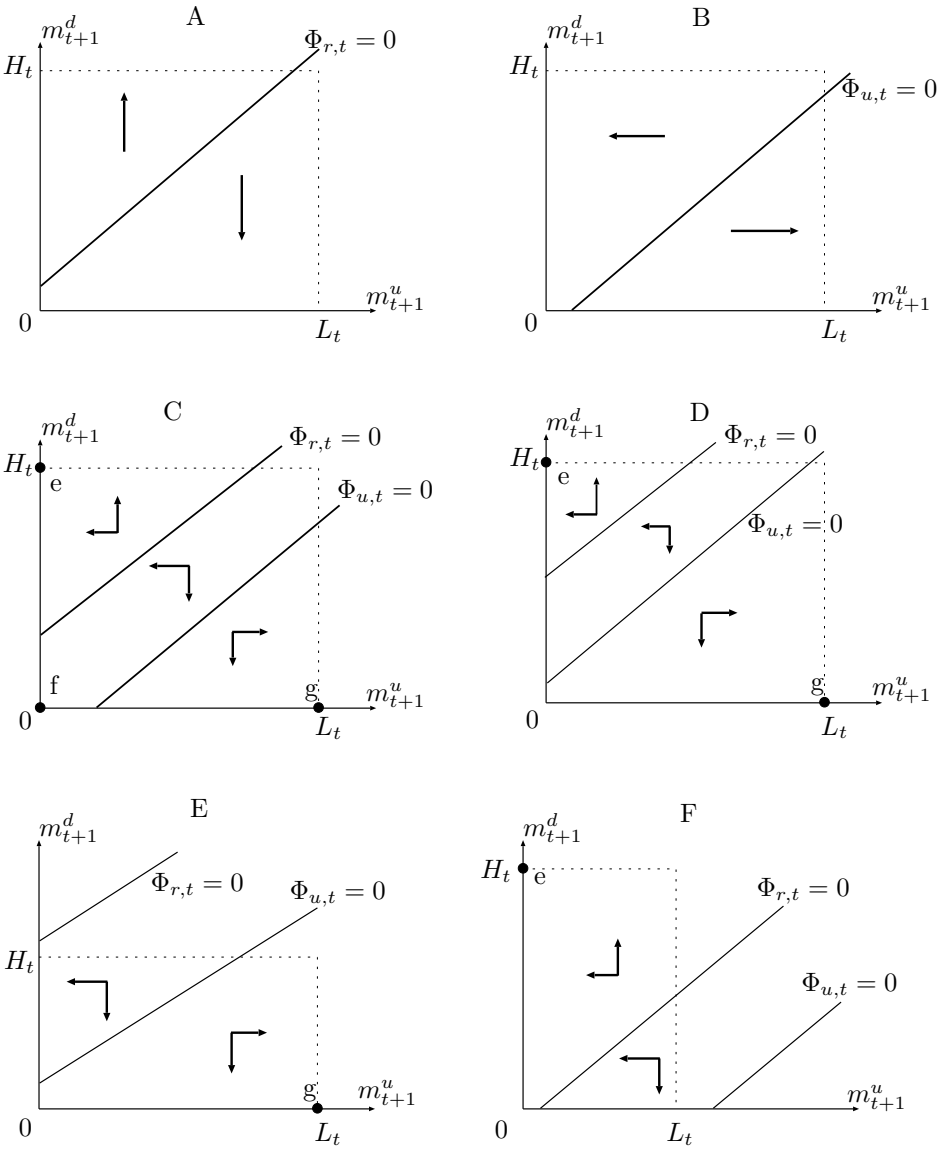

Figure 1: Utility premium of private education