Natural Disaster, Migration, and Regional Development†

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1. Introduction
In this paper, we extend a simple matching theory to consider how natural disasters affect regional economic activities and interregional migration.

In section 2, we introduce a simple model of matching theory based on the previous studies.1 This theory explains how unemployment rate, a measure of market tightness, wage rate, and other important variables are determined.

In section 3, we integrate the elements of natural disasters into the model of section 2. First, we assume agglomeration increases productivity. Second, natural disasters pull down production factors and thus deteriorate productivity. Then, population drain occurs.

In section 4, we extend the model of section 3. In 4.1, we consider regional loyalty. Damages caused by natural disasters decreases the utility of each household. However, suppose that the utility difference between domicile (hometown) and other regions are comparatively low. And so, it is taken for granted that people tend to stay in their hometown even if monetary gains becomes better off when they migrate to other areas. Under this premise, there are multiple steady states. In this case, natural disasters do not need to raise population drain.

In 4.2, we assume that productivity depends on public capital, which will be devastated by natural disasters. Just after the natural disaster, public capital decreases and people in this region may migrate to other regions. We also discuss the effects of fiscal policies to recover public capital. We show that once migration and a decline in population occur, such fiscal policies may deteriorate the regional economy. That is, excess supply of public capital increases the onus of the region and declines the utility of household. If so, fiscal policies may pose further population outflow.

2. Basic model of Matching
In this section, we show a simple matching model. The matching technology determines the total number of matches in the economy. Following the basic framework of matching theory (for example, Diamond, 1982; Mortensen, 1982; Pissarides, 1985), we specify the matching function as

\[ M = mu^\theta v^{1-\theta}, \]

where \( M \) is the total number of matches, \( U \) denotes the number of unemployed, \( V \) represents the number of vacancies, \( m \) and \( \alpha \) are the parameters (\( m > 0 \) and \( 0 < \alpha < 1 \)). Let us define \( \theta \equiv v/u \) as a measure of market tightness.

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The jobs are assumed to be broken at a rate $\lambda$ per period. Then the dynamic behavior of unemployment rate is given as

$$
\frac{du}{dt} = \lambda (1 - u) - mu^{\alpha},
$$

where $u = U/L$ and $v = V/L$ ($L$ denotes the number of population). In the steady state, the unemployment rate becomes

$$
u = \frac{\lambda}{(\lambda + m\theta^a)}.
$$

It is assumed that this economy has only one factor of production, labor. If a firm hires a worker, it can produce $y$ units of output and pay wage which is denoted as $w$. Each firm can earn net profit $(y - w)$ in every period until the match is dissolved. Let us represent the present discounted value of each firm that produces goods as $\Pi_e$, the present discounted value of a vacant job as $\Pi_v$, and the search cost for firm as $\delta$. Free entry condition means $\Pi_v = 0$. Then we obtain

$$
\Pi_e = \frac{(y - w)\gamma(\Pi_e - \Pi_v)}{r + \lambda} = \frac{\delta}{m\theta^a}.
$$

Equation (6) is regarded as a labor demand curve in the matching theory.

Let $V_e$ denote the representative discounted value of each employee, $V_u$ be the present discounted value of each unemployed person who searches a job. Bellman equations are given as

$$
r V_e = w + \lambda (V_u - V_e),
$$
$$
r V_u = b + m\theta^a (V_e - V_u).
$$

We assume that $w$ is determined endogenously through a process of bargaining between the worker and the firm (see Nash, 1953). In this paper, bargaining solution is to determine $w$ to maximize $(V_e - V_u)(\Pi_e - \Pi_v)^{-\gamma}$, where $\gamma$ is the bargaining power of the worker. Conditions for the maximum are given as $\gamma(\Pi_e - \Pi_v) = (1 - \gamma)(V_e - V_u)$. Equations (6), (7), and $\Pi_v = 0$ imply

$$
w = (1 - \gamma)\frac{b}{r} + \gamma \partial \Pi_v^*/\partial y + \gamma \delta \theta
$$

Equation (13) is regarded as a labor supply curve in the matching theory. In our model, equations (6) and (13) determine the wage rate and the measure of market tightness. Once $\theta$ is determined, we can derive the steady state values of $u$ and $v$. We can easily show that $\partial \Pi_v^*/\partial y > 0$, $\partial V_u^*/\partial y > 0$, and $\partial u^*/\partial y < 0$.

Furthermore, $V_u$ and $V_e$ are given as

$$
V_u = \frac{1}{r} \cdot \left\{ b + \gamma \theta^a (1 - \gamma) \right\},
$$
$$
V_e = \frac{1}{r} \cdot \left\{ b + \gamma \theta^a (1 - \gamma) \right\} \frac{1}{r + 1/m\theta^a + 1}.
$$

3. Simple model of Natural Disaster and Interregional Migration

From now on, we consider how natural disasters affect important variables such as per capita income,
population, unemployment rate and so on.

### 3.1 Marshallian Externalities

In this subsection, we focus on production function and utility. First, suppose that production function of firm \( j \) is given as \( Y_j = AL_j^\xi N_j \), where \( A \) and \( \xi \) are the parameters \((A > 0)\) and \((0 < \xi < 1)\), \( N_j \) represents the number of workers employed in firm \( j \), \( L_j \) denotes positive externalities from the regional population and \( L_j = L \) in equilibrium \((L \) is regional population). Note that we use the idea of Marshallian externality.\(^3\)

Each firm takes the value of \( L_j \) as given. Output per worker (which is denoted as \( y \)) is given as \( y = AL_j^\xi \).

So, labor productivity increases with regional population.

Let us describe households. If he is employed, his utility, \( W_e \) is given as \( W_e = V_e - h(L) \), where \( h(L) \) captures the negative externalities of congestion. We assume that \( h'(L) > 0, h''(L) > 0 \). If he is an unemployed person, his utility, \( W_u \) is represented as \( W_u = V_u - h(L) \).\(^4\)

From equations (6) and (13), and (14), it is shown that \( \partial V_e / \partial L > 0 \). Furthermore, we assume that \( \partial^2 V_e / \partial L^2 < 0 \). Then, the utility of unemployed \( (W_u) \) will be an inverted-U shape with respect to regional population, \( L \).

### 3.2 Population distribution before natural disaster

We assume that the common utility level of households is established for other regions. Let \( W' \) represent that common utility level. This view is similar to the open city model. This means that households that go to live in other regions can enjoy the welfare level, \( W' \). Households in a region consider this utility level \( W' \) as given. We draw this case in figure 4. Then we can obtain the following results:

1. If the population given in the initial stages is smaller than \( L_1 \) or larger than \( L_2 \), the households here have an incentive to migrate to other regions because the utility level established in this region is less than the \( W' \) provided in other regions.
2. If the population in this region is given as between \( L_1 \) and \( L_2 \), the households in other regions have an incentive to migrate to this region.
3. So, population in this region becomes 0 or \( L_2 \) in the long run.

We assume that population level is \( L_2 \) at time 0. If so, regional population is \( L_2 \) per capita output is given as \( y = AL_j^\xi \) for all \( t \) if natural disasters do not occur.

### 3.2 Population dynamics after natural disaster

Suppose that natural disaster occurs at time \( t \) and it devastates economic activities. It is assumed that per capita output becomes \( sAL_j^\xi \) rather than \( AL_j^\xi \), where \( 0 < s < 1 \). Labor productivity for a given value of regional population declines in consequence of natural disaster because productive factor is affected by

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\(^3\) Generally speaking, these positive externalities come from the number of employed worker rather than regional population. However, such setting does not affect our main results.

\(^4\) More precisely, the present discounted value of negative externalities is defined as \( h(L) = \int [e^{-rt}] h(t)L(t)dt \). Suppose that \( h_u'(L) > 0, h_u''(L) > 0, \) and \( L = L \) for all \( t \). Then, \( \int [e^{-rt}] h(t)L(t)dt = (1/r) h(L) \). Here, we define \( (1/r) h(L) = h(L) \).
natural disaster. In that case, per capita output as well as regional population decline. Per capita output declines through two channels. First, natural disaster alters production function. Second, a fall in population counteracts the positive effect of agglomeration. Output per worker decrease from $AL^z_2$ to $sAL^z_2$, where $L_2$ is as before and $L_3$ is given in figure 4. Note that this fall in population will increase an unemployment rate. The utility of representative unemployed remains unchanged because population drain alleviates congestion.

![Figure 4: the relationship between L and W_u after the natural disaster](image)

**4. Extensions of the model**

**4.1 Regional Loyalties**

In this subsection, we extend the model introduced in section 3. First, suppose that migrating to other regions involves some costs. This reflects regional loyalties, social capital that one has constructed in their life, moving costs including psychological burden, and so on. We write this cost as $F$. So, If households in this region migrate to other regions, they can enjoy the utility level denoted as $W' - F$ (we define $W'$ in section 3). Households in other regions have an incentive to migrate to this region if the utility level of unemployed in this region is higher than $W' + F$. We maintain other assumptions that we made in the previous section. Per capita output is given as $AL^z$ before the natural disaster. If a natural disaster strikes, then per capita output becomes $sAL^z$ instead of $AL^z$.

First of all, we will focus on the case of before the natural disaster (see figure 6).

1. If the population given in the initial stages is smaller than $L_1$, the households here have an incentive to migrate to other regions and this region disappears in the long run.
2. If the initial value of the regional population is larger than $L_6$, regional population converges to $L_6$.
3. If the population in this region is given as between $L_4$ and $L_3$, or between $L_4$ and $L_6$, the population remains unchanged.
4. If the population in this region is given as between $L_3$ and $L_4$, regional population converges to $L_4$.

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5 We implicitly assume the existence of production factors other than labor. Presuming that the parameter $A$ depends on these factors, it is natural to consider $A$ becomes $sA$ by natural disasters. In section 4, we will consider this point in detail.
So, regional population becomes between $L_1$ and $L_3$ or between $L_4$ and $L_6$ in the steady state. In the model we consider in section 3, the steady state values of regional population are 0 or $L_2$ in figure 3. However, if we consider the term ‘regional loyalties’, there are lots of steady states.

Suppose that natural disaster occurs at some date and production function moves to $sAL^\xi$. Then utility curve shifts downward (see figure 6) and we obtain the following results:

(5) If the population before the natural disaster is between $L_1$ and $L_3$, regional population converges to 0. That is, if population before the natural disaster is relatively small, the natural disaster makes it impossible for the afflicted region to maintain the economic activities.

(6) If the population in this region before the natural disaster is given as between $L_2$ and $L_5$, or $L_4$ and $L_5$, Regional population remains unchanged.

(7) If regional population before the natural disaster is larger than $L_5$, post-disaster regional population converges to $L_5$.

Note that the utility level decreases after the natural disaster unless the initial level of population is $L_5$. In section 3, natural disaster decreases regional population, although the utility level of unemployed remains unchanged. However, in the extended model here, not only population distribution but also utility level will become altered. The unemployment rate will increase because natural disaster lowers labor productivity.

4.2 Production Function with Infrastructure

In section 3, we have assumed that per capita output, $y$, is given as $y = AL^\xi$. Here, we introduce another production function. Suppose that output per worker is defined as $y = AG^\beta$, where $G$ is public capital or,
infrastructure. We assume that public capital is provided by the central government and maintenance is undertaken by the local government. So, the local government takes the values of $G$ as given.

To maintain $G$ units of public capital which is provided by a central government, the local government must collect $\varepsilon G$ units of final good in every period. We assume that the local government imposes a tax on firms and each firm must incur $\tau$ units of output. Budget constraint of the local government is given as $\tau (1 - u^*) L = \varepsilon G$. Let us call $(y - \tau)$ as net output per worker.

Some assumptions insure the inverted U shaped relationship between regional population and the utility of an unemployment person. So, we can use figure 6 to analyze the interregional migration.

The Natural disaster occurs at some date and public capital is devastated. Suppose that public capital becomes $s'G$ rather than $G$ in the aftermath of the natural disaster. We assume that before the natural disaster, the regional population, $L^0$, is between $L_1$ and $L_2$ or between $L_4$ and $L_6$.

The effects of the natural disaster are similar to those of section 4.1. Hereafter, we mainly focus on the case where $L_5 < L^0 < L_6$. As discussed above, natural disaster decreases regional population from $L^0$ to $L_5$.

Next, we analyze how fiscal policies to recover public capital destroyed by the natural disaster affect people's economic activities. Suppose that the government promotes public investment to recover public capital to $G^*(L^0)$ from $s'G^*(L^0)$. However once regional population becomes $L_5$, $G^*(L^0)$ is no longer optimal. If regional population is $L_5$, and $\partial Y^*/\partial L < 0$ at $L=L_5$, the reconstruction plans for public investment accelerate the population drain. If $\partial Y^*/\partial L > 0$, public investment to recover public capital may increase the net output. In this case, the utility of residents in the region will increase. However, once people migrate to the other regions, it may be hard to increase the number of households to the original level.

**Main References**


