Capital-Intensity Hypothesis and Factor Price Equalization Theorem: Intriguing Relationship?

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Prepared for RSIJ Annual Conference at Tokushima University, October 14, 2013

ABSTRACT: Predicated upon a 2x2x2 model of general equilibrium, this paper shows conditions under which the so-called factor price equalization theorem may not hold. Consider a developing nation with a strong propensity to save and accumulate capital on one hand and a developed country that can afford specializing in consumption on the other hand. If the consumption good sector is more capital-intensive than the investment sector, then free trade can lower the developing nation's wage rate while increasing the developed nation's wage rate. The assumed conditions are all empirically plausible, and the outcome sounds surprising, and deserves theoretical scrutiny.

Key Words:

2x2x2 general equilibrium, capital intensity, factor price equalization, trade, capital-rich *versus* -poor countries

Introduction

According to the factor price equalization theorem real factor prices such as relative wage rates tend to equalize among nations after trade. But this well-known theorem may not hold, if the less well-known hypothesis on certain particular technological conditions holds true. The latter, called the capital-intensity hypothesis (R.M. Solow, 1956; Y. Shinkai, 1960), assumes two sectors of production, and states that stable economic growth requires, though not necessarily, the consumption good sector to be more capital intensive than the investment good sector. This condition has recently been confirmed empirically by H. Takahashi, *et. al*. (2012) utilizing data from the IO tables of the OECD countries, and H. Kawano, *et. al*. (2009, 2012) using a simple applied general equilibrium model.

The present paper challenges the factor price equalization theorem on the basis of an empirically plausible condition, albeit less well known, on capital intensities in consumption and investment sectors. We show that if a rich developed nation endowed with large capital and a poor developing country with small capital trade their products, then their real factor prices may tend to diverge rather than converge.

The crucial conditions required for such a startling outcome other than the capital-intensity hypothesis aforementioned are that the developing nation being capital-poor has a strong taste for investment goods, and the developed capital-rich nation specializes in consumption. *The poor stay hungry, and the rich remain foolish.*

Background

A paper by H. Ohta (2004) has revealed certain moral scientific nature of the well-known Stolper-Samuelson (SS) theorem and the related less well-known proposal to 'bribe' for free trade. Ohta assumed two identical countries, identically endowed with homogeneous labor and capital (two factors) but with internationally different tastes. Pursuant to these basic assumptions he presented a simple account of the SS theory in a simple formal 2x2x2

general equilibrium framework. The H.O. (2005) presented his heretical ideas on different tastes in addition to the orthodox HO ideas on different endowments. It in turn inspired H.O. to ponder on R. Jones and R. Ruffin (2005, 2007) discussing their "technology transfer paradox" along with some other seeming paradoxes in trade theory in H.O. (2006), elaborated further with Hiro Nakagawa (*APJAE*, 2008). The more recent H. O. (2009, 11, 12) are getting more directly focused on a **bitter maxim** of free trade that the Stolper-Samuelson theorem revealed.

With these prior inquiries in the background the present paper goes beyond the static model of general equilibrium of production and exchange by incorporating in it capital formation and economic growth. It shows conditions under which how factor prices, product prices, and sectoral factor intensities may be related to the stages of economic growth. If the bitter maxim of free trade is an inconvenient truth despite the factor price *equalization* theorem, the bitterness may intensify all the more if and when the rosy theorem is reversed.

In what follows Section 1 reviews the basic 2x2 general equilibrium model of two clone nations with no growth, nor trade. The model for a representative clone country is presented using just one CD parameter to represent both technology and taste. Section 2 in turn presents a 2x2x2 by 1 CD parameter model of two contrasting nations with asymmetric national tastes before and after trade to examine what impact free trade may have on factor prices and commodity prices. We confirm the orthodox outcomes of free trade and the related factor-price equalization theorem. Section 3 then introduces growth, asymmetric growth in particular, to see what happens to factor prices between the trading nations. The question is if factor price equalization will be achieved. The answer is, no. Moreover, factor prices that may be equal before trade will necessarily diverge after trade. Section 4 concludes.

Section 1. The 2x2 GE model of clone economies before asymmetric growth and trade

Consider an autarky economy endowed with fixed amounts of two factors: labor and capital. These factors are used to produce consumption goods and investment goods under constant returns to scale, of the Cobb-Douglas type:

The Consumption Good Sector: $C = K_C^{\alpha} L_C^{(1-\alpha)}$

The Investment Good Sector: $I = K_I^{(1-\alpha)} L_I^{\alpha}$

where C stands for consumption, I for investment, α 'output elasticity' of capital in the C sector and also 'output elasticity' of labor in the sector I. This is a deliberate contrivance to differentiate the two sectors' methods of production by a single parameter α.

Now in order are the following optimization problems and equilibrium conditions.

Consumptive Optimum:

$$U_1(I, C)/p = U_2(I, C) \qquad (1)$$

where U_1 is MU of the first good, I, U_2 MU of the second good C, and p the relative price of investment goods I in terms of consumption goods C assumed as the *numeraire*.

Individual Budget Constraints:

$$w = pI_L + C_L;\ r = pI_K + C_K, \qquad (2), (3)$$

Productive Optimum:

$w/r = f_2/f_1 = g_2/g_1$, (4), (5)

where w is wage rate, r rent on capital, f_1(or g_1) is the C (or I) sector's MP of capital, f_2 ($=f(k_I,1)-k_If_1(k_I,1))=\phi(k_I)$ is the I sector's MP of labor, which is a function of capital/labor ratio or capital intensity in the I sector, and g_2($=g(k_C, 1)-k_Cg_1(k_C, 1))=\gamma(k_C)$ is MP of labor in the C sector, as a function of C sector's capital/labor ratio.

Factor Market Equilibrium:

$K_I + K_C = 1$; $L_I + L_C = 1$ (6), (7)

Product Market Equilibrium:

$I = f(K_I, L_I)$; $C = g(K_C, L_C)$ (8), (9)

Walras' Law:

$pI + C = w + r$ (10)

Equi-MRTS (of *L* for *K*) for Optimal Resource Allocation

The following optimization conditions are to be observed along a contract curve, which is concave when $\alpha > 1/2$.

$(-dK_I/dL_I) = ((1-\alpha)/\alpha)(K_I/L_I) = (-dK_C/dL_C) = (\alpha/(1-\alpha))(K_C/L_C)$

Combined with the conditions of a given endowment of L and K, this yields.

$$K_C = \frac{1}{(\frac{1}{L_C} - 1)(\frac{1-\alpha}{\alpha})^2 + 1} \qquad (\boldsymbol{K = L = 1})$$

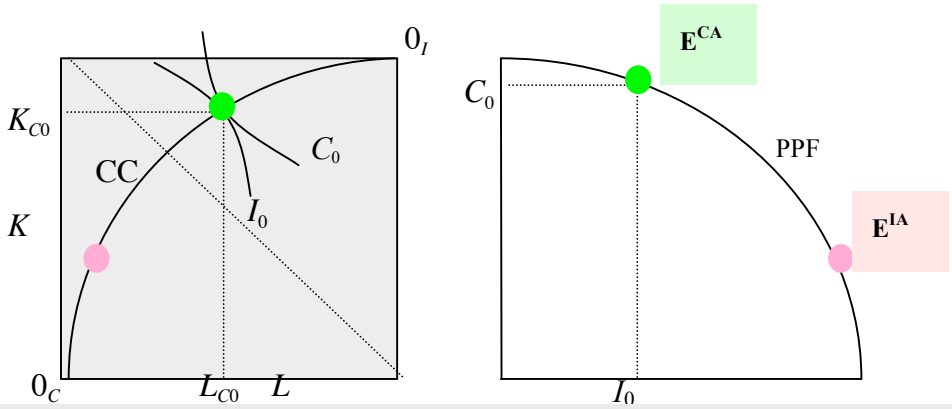

Figure 1. Given α, L_I determines K_I, hence (L_I, K_I) as a one-to-one correspondence with (C, I), and w/r

Figure 1 illustrates an autarkic equilibrium, labeled $\mathbf{E}^{\mathbf{CCA}}$, for country **C**, and $\mathbf{E}^{\mathbf{CIA}}$ for Country **I** on the conflict curve CC on the left and the production possibility frontier on the right. Here Country **C** produce and consume more consumption goods than the **I** do. The **C** workers, before trade, are poorer than the capitalists. Not only are the **C** workers poorer than the **C** capitalists, but also they are poorer than the **I** workers. The **I** workers, by comparison, are richer than not only their domestic capitalists, but also the **C** workers abroad.

These observations are to be made on Figure 2 below by comparing a square point with a triangular point along the conflict curve on the left. Wage rate given by factor allocation identified at the triangular point must be strictly greater than unity and hence greater than at the square point, which is less than unity. Related to this

particular wage rate of unity is a circular point at which MRTS is required to be unity. Not only wage rate is unity at this point, but also so is product price identified at a circular point on the production possibility frontier on the right.

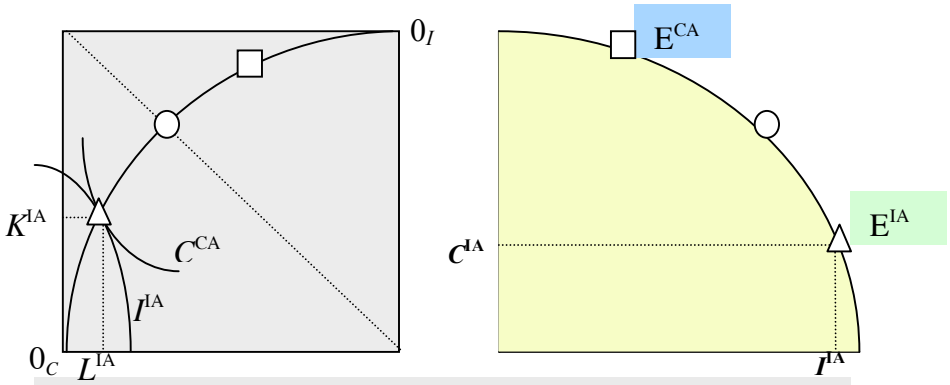

Figure 2. Alternative resource allocations and related autarky equilibria

Underlying the diagram above is a formal 2x2x2 general equilibrium model of production, trade, and consumption.

Section 2. 2x2x2 GE of clone economies with hetero-national tastes before/after trade

(Omitted)

Section 3. Impact of trade under asymmetric growth and tastes upon factor prices

Despite the bitter outcomes of trade both nationals ***C*** and ***I*** each as a nation are richer after trade, and the **C** can consume more consumption goods while the **I** can invest more than they could before trade. This implies that while the factor endowment in **C** remains the same regardless of trade the capital endowment in **I** tends to increase over time, all the more after trade. As the **I** grows to be a developed economy, it can afford turning itself into a country specializing in consumption. The ex-**I** is now a developed country **DC**. The ex-**C** realizing no growth, by comparison, may learn from the ex-**I** to turn itself to a new developing country **DI**.

What then would happen to factor prices in these countries after trade? Would they equalize between **DC** and **DI**? To answer this question we restate formally the following basic assumptions/conditions of our analysis.

A1) Two clone countries exist with divergent national tastes: the one addicted to consumption, called **DC**, and the other to saving and investment, called **DI**.

A2) Each country is endowed with two factors of production, capital K and labor L, to produce two kinds of goods, one for consumption and the other for investment: C and I.

A3) In both countries labor is owned by a given number of workers L, and capital by a given number of capitalists K, in the same number, say, L=K=1. However, as ex-**I** grows capital accumulated K^* becomes distinctively higher than in the ex-**C** so that $K^* > K=1$ in the long run or asymmetric growth, say, $K^*_{\text{x-CI}} = 4$ (for contrasting purposes only)

A4) Each country has two sectors of production: one for consumption. another one investment goods.

A5) The C sector is more capital-intensive than the I sector. (This requires $\alpha > 1/2$.)

A6) The developing country is investment-oriented: producing investment goods extensively with little

consumption goods, therefore living on thin air. The poor stay hungry.

A7) The developed country is *more consumption- than investment-oriented*. They specialize in consumption taking advantage of rich capital while producing little investment goods.

These specifications of the Assumptions above are needed only for the sake of yielding the initial conditions for autarky equilibrium for the two nations' CPs and FPs before trade as illustrated by Figure 5 below.

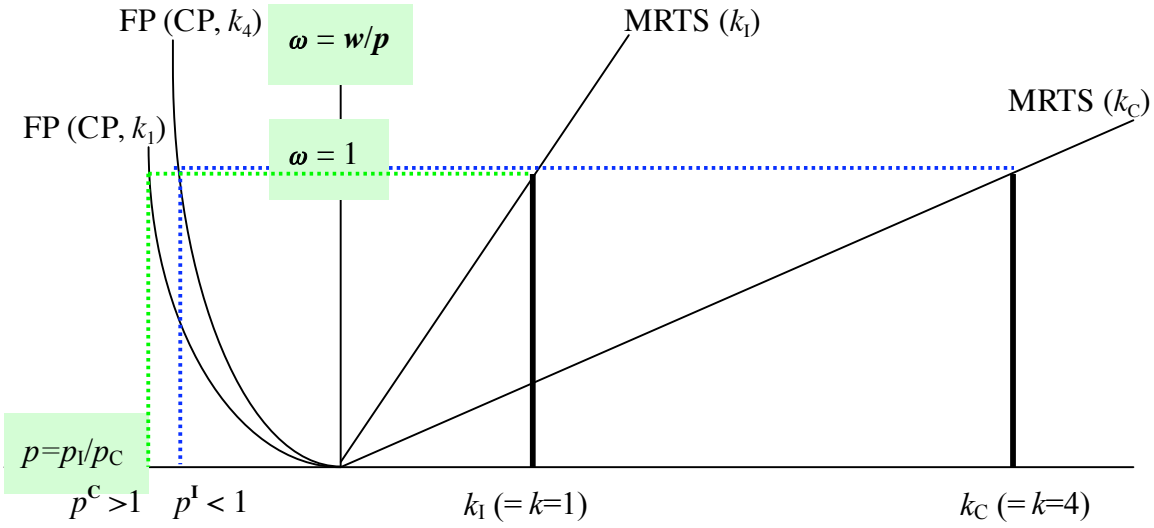

Figure 5. Sectoral k_i's, and aggregate *ks* that yield equal FPs (=1), yet distinct CPs

The differences in CPs in two countries provide apparent incentives for them to trade. The question then is what happens to prices CPs and FPs. Remember that given $k = 1$ for two clone countries trade tends to equalize them all, as illustrated by Figure 4 *supra*, along which the high prices must fall and the low prices rise after trade.

But in the long run FPs could diverge insofar as capital accumulation proceeds in Country **I** where the workers may tend to resist opening their market in fear of their real wage declines at least in the short run when k is fixed. But *over time* while the stagnant **C**'s factor endowment remains unchanged the **I**'s capital must increase. Moreover, free trade will accelerate their capital accumulation, thereby increasing their wages over time to such an extent that their real wage will exceed the stagnant **C**'s when capital accumulation proceeds beyond $k = 4$. The growing **I**'s real wages will also increase with increasing output of I to be exported by the stagnant **C** to the growing **I**, but will peak out when it reaches ω (=w/r) 2 in the figure. Thus, real wages of a growing economy can surpass those in the stagnant economy, all the more with trade, accelerating capital accumulation.

Further accumulation beyond $k = 4$ keeps price of investment goods increasing as the ex-**I** can now specialize in consumption on their production possibilities frontier that has kept shifting outward with increasing slopes at the C axis. They are now on $\mathbf{PPF}^{\mathbf{DC}}$ and capable of offering the entire amount of the C in exchange for the maximum I the ex-**C** can produce along $\mathbf{PPF}^{\mathbf{DI}}$ that intersects with the I axis. The maximum wage $\omega^{\mathbf{DI}}{}_{max}$ that the **DI** can earn by producing the investment goods at $\mathbf{E}^{\mathbf{DIP}}$* is now strictly lower than the minimum $\omega^{\mathbf{DCC}}{}_{min}$ that the **DC** earn by specializing in consumption now at the left corner of $\mathbf{PPF}^{\mathbf{DC}}$ with $\boldsymbol{k}^{\mathbf{DI}} > 4$. See Figure 6.

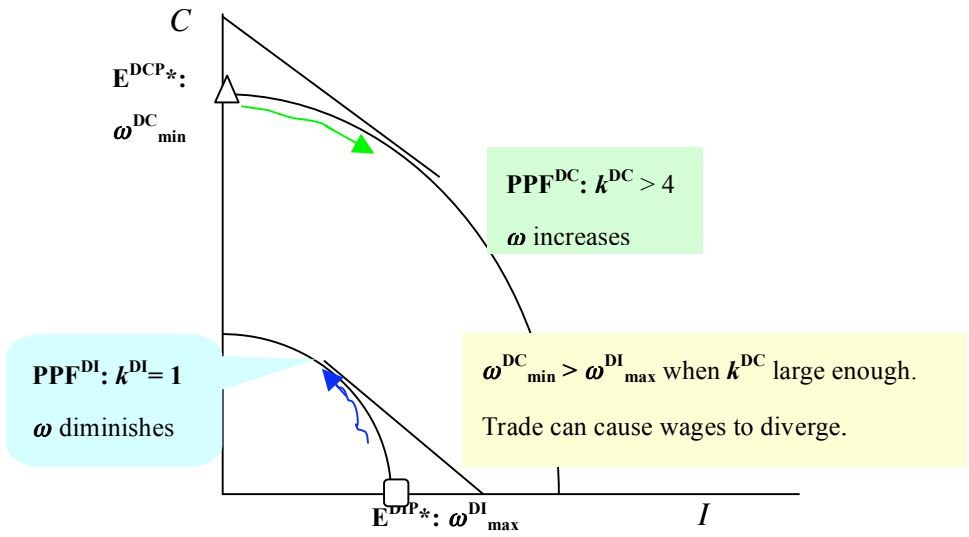

Figure 6-1. Trade can cause factor prices to diverge

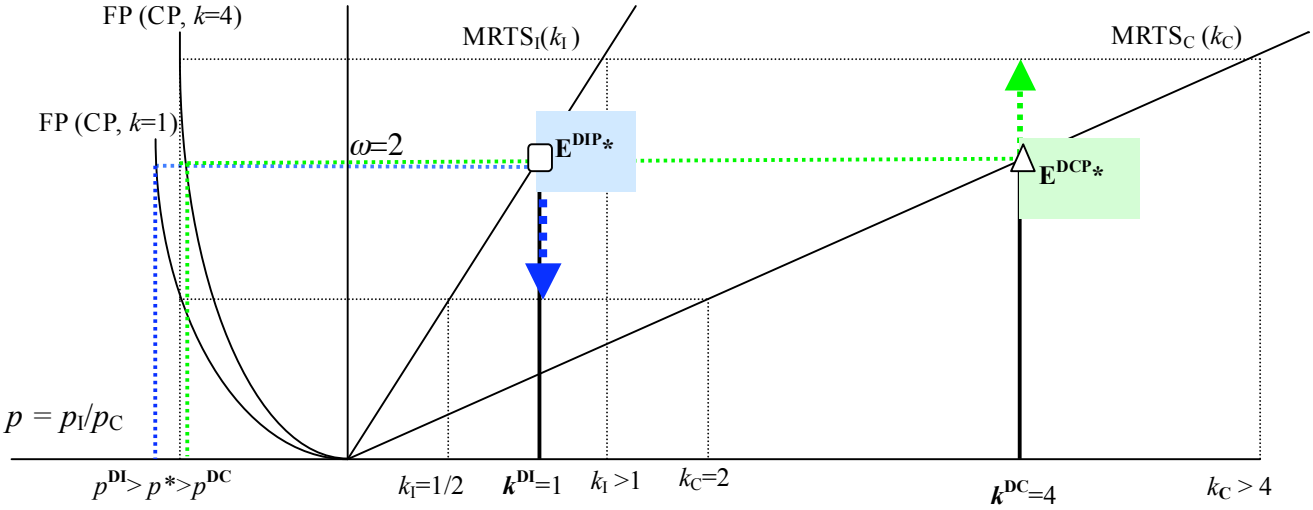

Figure 6-2. Trade can cause factor price diversions under asymmetric tastes and growth

Underlying Figure 6 are assumptions **A1)** through **A7)**. Figure 6-1, upper panel, depicts two contrasting production possibility frontiers $\mathbf{PPF}^{\mathbf{DI}}$ and $\mathbf{PPF}^{\mathbf{DC}}$ of the ex-developing (now rich) **DC** and the ex-consumption-addicted (now hungry) country **DI**, respectively. Note initially that asymmetric autarky equilibria are represented by $\mathbf{E}^{\mathbf{DIP}}*$ and $\mathbf{E}^{\mathbf{DCP}}*$ and related equilibrium commodity prices of $\boldsymbol{p}^{\mathbf{DC}} > \boldsymbol{p}^{\mathbf{DI}}$.

Then the two arrows show how trade may yield resource reallocations and related changes in the commodity bundles produced and, of particular interest to the present inquiry, the factor prices between the countries under consideration. The rich **DC** that now prefers consumption to investment nevertheless had better produce less consumption goods and more investment goods for export while the poor **DI** that now prefers investment to consumption may produce more consumption goods for export. Thus they move along their own **PPC**s until when $\boldsymbol{p}^{\mathbf{DC}}=\boldsymbol{p}^{\mathbf{DI}}$ is established. In this process toward equilibrium what happens to factor prices within the trading countries is that they must necessarily diverge as indicated by the dotted arrows in opposite directions.

Moreover, trade lowers the poor nation's real wages and increases the rich nationals' wage income! An inconvenient truth would it be? Would it then call for the SS proposal to 'bribe' for free trade with no apology? H.O. disagrees.

Section 4. Conclusions

A simple model of general equilibrium with trade by H. Ohta (2004, 5, 6, 11, 12) has been revisited to probe the Stolper-Samuelson theorem and the related orthodoxy of the HO factor price equalization theorem. We find that free trade may dis-equilibrate factor prices under certain particular empirically plausible conditions of asymmetric international tastes among nations at the outset of asymmetric growth, followed by a plausible shift in, or reversal of national tastes upon asymmetric growth. Taste reversal is likely to occur insofar as an ex-hungry, now rich nation can afford specializing in consumption while a stagnant nation with little prior saving realizes that they need to become hungry to take off for growth/development.

This conclusion, if surprising, can be confirmed by the departure point conditions, that factor prices, say, real wages, happened to be equal in a poor developing country and a rich developed country under autarky, respectively. Within the confines of a simple 2x2x2 GE model presented in Section 3, related Figures 4 - 6, this requires capital stock of the developed country to be four times that of the developing country. The assumed technological clones are represented by just one CD parameter $\alpha = 2/3$. This last specification reflects our other related assumption that requires the consumption sector to be more capital intensive than the investment sector.

This required condition is crucial to our present inquiry, crucial for two counts. First, our conclusion above crucially depends upon this seemingly unrealistic assumption. Second, if this seemingly unrealistic assumption were in fact unrealistic, then our surprising conclusion must collapse. Fortunately, the seemingly unrealistic assumption is empirically supported, and hence crucial and relevant in the second sense. And our conclusion is valid.

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